Multimea numerelor reale

$\mathbb{N} = \{0, 1, 2, 3, \dots, n, \dots\}$ multimea nr. naturale

$\mathbb{Z} = \{\dots -n, \dots, -3, -2, -1, 0, 1, 2, 3, \dots, n, \dots\}$ multimea nr. intregi

$\mathbb{Q} = \left\{ \frac{m}{n} \mid m, n \in \mathbb{Z}, n \neq 0 \right\}$ multimea nr. rationale

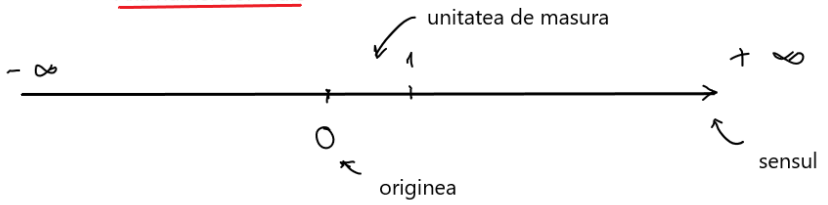
$\bar{\mathbb{Q}} = \{$ nr. care NU pot fi scrise ca fractii $\}$ multimea nr. irationale

$\mathbb{R} = \mathbb{Q} \cup \bar{\mathbb{Q}}$ multimea nr. reale

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

$\bar{\mathbb{Q}} \cup$

Axa numerelor reale



Exista o corespondenta 1-1 intre multimea numerelor reale si multimea punctelor dreptei reale; adica fiecarui nr. real ii corespunde un singur punct de pe dreapta si invers fiecarui punct de pe dreapta ii corespunde un singur nr. real.

Partea fractionara a numarului real x este numarul pozitiv dat de diferenta $x - [x]$.

Partea fractionara se noteaza $\{x\}$.

Prin urmare avem relatia $x = [x] + \{x\}$, $\forall x \in \mathbb{R}$.

Ex: $x = 1,25$

$[x] = 1$, $\{x\} = x - [x] = 1,25 - 1 = 0,25 > 0$

$x = 1,25 = 1 + 0,25 = [x] + \{x\}$

Ex: $x = -1,25$

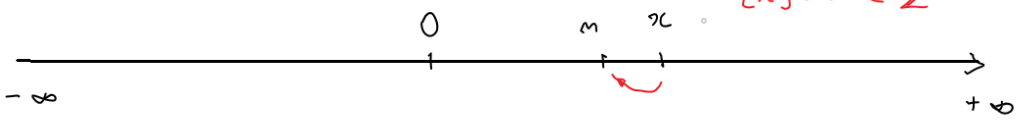
$[x] = -2$, $\{x\} = x - [x] = -1,25 - (-2) = -1,25 + 2 = 0,75 > 0$

$x = -1,25 = -2 + 0,75 = [x] + \{x\}$

Partea intreaga si partea fractionara a unui numar real

$x \in \mathbb{R}$

$[x] = m \in \mathbb{Z}$



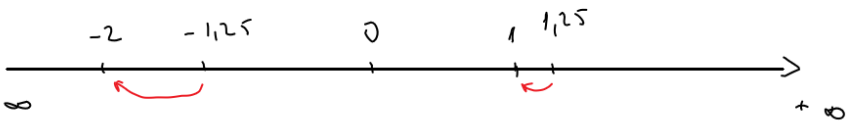
Partea intreaga a unui numar real x este primul numar intreg de la stanga pe axa numerelor reale.

Daca $x = 1,25$

$[x] = [1,25] = 1$

Daca $x = -1,25$

$[x] = [-1,25] = -2$



Notatie: $[x]$ partea intreaga a lui $x \in \mathbb{R}$

Aplicatii

Sa se calculeze partea intreaga si partea fractionara a numerelor

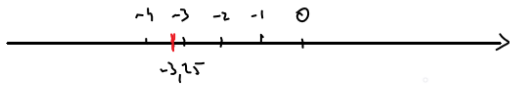
a) $x = 7,3 = 7 + 0,3 \Rightarrow [7,3] = 7, \ \{7,3\} = 0,3$

b) $x = 2,1(8) = 2 + 0,1(8) \Rightarrow [2,1(8)] = 2, \ \{2,1(8)\} = 0,1(8)$

c) $x = -3\frac{1}{4} = -\left(3\frac{1}{4}\right) = -\left(3 + \frac{1}{4}\right) = -(3 + 0,25) = -3,25 \Rightarrow [-3\frac{1}{4}] = -4$

d) $x = -\frac{425}{12}$

e) $x = \frac{2020}{14}$



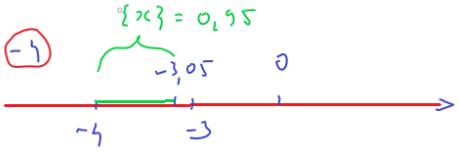
$\{-3,25\} = -3,25 - (-4) = -3,25 + 4 = 0,75$

Aplicatii

a) $x = 2,35 \Rightarrow [x] = 2, \ \{x\} = \{2,35\} = 0,35$

b) $x = -3,05 \Rightarrow [x] = [-3,05] = -4$

$\{x\} = x - [x] = -3,05 - (-4) = 0,95$



c) $x = \frac{7}{2} = 3,5$

$\Rightarrow [x] = [3,5] = 3, \ \{x\} = x - [x] = 3,5 - 3 = 0,5$

TEMĂ

Să se determine $[x]$ și $\{x\}$ dacă $x \in \{2,01; -3,2; -4,102; 7,3\}$

d) $x = -\frac{425}{12} = -31,41(6) = -(31 + 0,41(6)) = -31,41(6)$

$\left[-\frac{425}{12}\right] = -32$, $\left\{-\frac{425}{12}\right\} = -\frac{425}{12} - (-32) =$

$= -31,41(6) + 32 = -31 - 0,41(6) + 32 =$

$= 1 - 0,41(6) = 0,58(3)$

e) $x = \frac{2020}{14} = 144,(285714)$

$\left[\frac{2020}{14}\right] = 144$, $\left\{\frac{2020}{14}\right\} = \frac{2020}{14} - 144 = \frac{2020 - 2016}{14} = \frac{4}{14} = \frac{2}{7} =$

$= 0,(285714)$

OPERAȚII CU NUMERE REALE

$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$

$(a-b)^2 = (a-b)(a-b) = a^2 - ab - ba + b^2 = a^2 - 2ab + b^2$

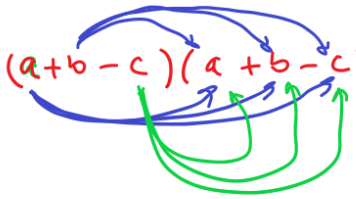
$(a-b)(a+b) = a^2 + ab - ba - b^2 = a^2 - b^2$

$(a+b)^3 = (a+b)(a+b)^2 = (a+b)(a^2 + 2ab + b^2) = a^3 + 2a^2b + ab^2 +$

$+ ba^2 + 2ab^2 + b^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = (a-b)(a-b)^2 = (a-b)(a^2 - 2ab + b^2) = a^3 - 2a^2b + ab^2 -$

$- ba^2 + 2ab^2 - b^3 = a^3 - 3a^2b + 3ab^2 - b^3$



$(a+b-c)(a+b-c) = a\cdot a + a\cdot b - ac + b\cdot a + b\cdot b - bc$
$- (a\cdot c + c\cdot b - c\cdot c) =$

$= a^2 + ab - ac + ab + b^2 - bc -$
$- (ac + cb - c^2) =$

$= a^2 + 2ab - ac - bc - ac - cb + c^2 + b^2$

$= a^2 + b^2 + c^2 + 2ab - 2ac - 2bc$

$(a+b-c)^2 - 2ab + 2ac + 2bc = a^2 + b^2 + c^2 + 2ab - 2ac - 2bc -$
$- 2ab + 2ac + 2bc$
$\Rightarrow a^2 + b^2 + c^2$

APLICAȚII

$(3x+1)^2 - (x+3)^2 = a^2 - b^2 = (a-b)(a+b) =$
($a = 3x+1$, $b = x+3$)

$= (3x+1-(x+3))(3x+1+x+3) =$

$= (3x+1-x-3)(4x+4) =$

$= (2x-2)(4x+4) = 2(x-1)\cdot 4(x+1) =$

$= 8(x-1)(x+1) = 8(x^2-1) = 8x^2 - 8$

$x^2 - 2\sqrt{3}x + 3 - (y-2\sqrt{3})^2 = (x-\sqrt{3})^2 - (y-2\sqrt{3})^2 = a^2 - b^2 = (a-b)(a+b)$
($a = x-\sqrt{3}$, $b = y-2\sqrt{3}$)

$= (x-\sqrt{3}-(y-2\sqrt{3}))(x-\sqrt{3}+y-2\sqrt{3}) =$

$= (x-\sqrt{3}-y+2\sqrt{3})(x-\sqrt{3}+y-2\sqrt{3}) =$

$= (x-y+\sqrt{3})(x+y-3\sqrt{3})$

$(\underbrace{3x}_{a} \underbrace{-2y}_{b} + \underbrace{\sqrt{5}}_{c})^2 = a^2+b^2+c^2+2ab+2bc+2ac =$

$= (3x)^2 + (-2y)^2 + (\sqrt{5})^2 + 2\cdot 3x(-2y) + 2(-2y)\sqrt{5} + 2\cdot 3x\cdot\sqrt{5} =$

$= 9x^2 + 4y^2 + 5 - 12xy - 4\sqrt{5}y + 6\sqrt{5}x$

$(a-b)(a^2+ab+b^2) = a^3 + a^2b + ab^2 - ba^2 - ab^2 - b^3 =$

$= a^3 - b^3$

$(a+b)(a^2-ab+b^2) = a^3 - a^2b + ab^2 + ba^2 - ab^2 + b^3$

$= a^3 + b^3$

$(\underbrace{2\sqrt{2}}_{x} \underbrace{-3a}_{y})(\underbrace{2\sqrt{2}}_{x} \underbrace{+3a}_{y}) = (x-y)(x+y) = x^2 - y^2 = (2\sqrt{2})^2 - (3a)^2 =$

$= 4\cdot 2 - 9a^2 = 8 - 9a^2$

$(a+b+c)^2 = (a+b+c)(a+b+c) = a^2 + ab + ac + ba + b^2 +$

$+ bc + ca + cb + c^2 = a^2+b^2+c^2+2ab+2bc+$

$+ 2ac$

APLICAȚII

$(3x+1)^2 - (x+3)^2 = a^2 - b^2 = (a-b)(a+b) =$ ($a = 3x+1$, $b = x+3$)

$= (3x+1-(x+3))(3x+1+x+3) =$

$= (3x+1-x-3)(4x+4) =$

$= (2x-2)(4x+4) = 2(x-1)\cdot 4(x+1) =$

$= 8(x-1)(x+1) = 8(x^2-1) = 8x^2 - 8$

$x^2 - 2\sqrt{3}x + 3 - (y-2\sqrt{3})^2 = (x-\sqrt{3})^2 - (y-2\sqrt{3})^2 = a^2 - b^2 = (a-b)(a+b)$ ($a = x-\sqrt{3}$, $b = y-2\sqrt{3}$)

$= (x-\sqrt{3}-(y-2\sqrt{3}))(x-\sqrt{3}+y-2\sqrt{3}) =$

$= (x-\sqrt{3}-y+2\sqrt{3})(x-\sqrt{3}+y-2\sqrt{3}) =$

$= (x-y+\sqrt{3})(x+y-3\sqrt{3})$

Aplicații:

$(2x-y)^2 = (a-b)^2 = a^2 - 2ab + b^2 = (2x)^2 - 2(2x)\cdot y + y^2 =$ ($a = 2x$, $b = y$)

$= 4x^2 - 4xy + y^2$

$(2\sqrt{2}-a)(2\sqrt{2}+a) = (x-y)(x+y) = x^2 - y^2 = (2\sqrt{2})^2 - a^2 = 8 - a^2$ ($x = 2\sqrt{2}$, $y = a$)

$\left(\frac{x}{3}+2y\right)^2 = (a+b)^2 = a^2 + 2ab + b^2 = \left(\frac{x}{3}\right)^2 + 2\cdot\frac{x}{3}\cdot 2y + (2y)^2 =$ ($a = \frac{x}{3}$, $b = 2y$)

$= \frac{x^2}{9} + \frac{4}{3}xy + 4y^2$

$\frac{5}{11} = 0,(45)$ $\qquad$ $0,(45) = \frac{45}{99} = \frac{5}{11}$

$50 : 11 = 0,4545\ldots = 0,(45)$

-44

$= 60$

-55

$= 50$

-44

$= 60$

-55

$= 5$

$6+5x = 2(-3+x) \Leftrightarrow 6+5x = -6+2x \Leftrightarrow 5x-2x = -6-6 \Leftrightarrow$

$\Leftrightarrow 3x = -12 \mid :3 \Leftrightarrow x = -12:3 \Leftrightarrow \boxed{x = -4}$

$x+18 = 2(-1-x)+8 \Leftrightarrow x+18 = -2-2x+8 \Leftrightarrow x+2x = -2+8-18$

$\Leftrightarrow 3x = -12 \mid :3 \Leftrightarrow x = -12:3 \Leftrightarrow \boxed{x = -4}$

Aceeași soluție $x = -4$, deci sunt echivalente.

$x = \sqrt{300} + 2\sqrt{192} - 3\sqrt{675} - \sqrt{108} =$

$= 10\sqrt{3} + 2 \cdot 8\sqrt{3} - 3 \cdot 15\sqrt{3} - 6\sqrt{3} =$

$= \sqrt{3}(10 + 16 - 45 - 6) = (26 - 51)\sqrt{3} =$

$= \boxed{-25\sqrt{3}}$

$\sqrt{300} = \sqrt{3 \cdot 100} = \sqrt{3} \cdot \sqrt{100} = 10\sqrt{3}$

$\sqrt{192} = \sqrt{3 \cdot 64} = \sqrt{3} \cdot \sqrt{64} = 8\sqrt{3}$

$\sqrt{675} = \sqrt{3 \cdot 225} = \sqrt{3} \cdot \sqrt{225} = 15\sqrt{3}$

$\sqrt{108} = \sqrt{3 \cdot 36} = \sqrt{3} \cdot \sqrt{36} = 6\sqrt{3}$

$y = \sqrt{432} + 2\sqrt{588} - 3\sqrt{1200} + 15\sqrt{12} =$

$= 12\sqrt{3} + 2 \cdot 14\sqrt{3} - 3 \cdot 20\sqrt{3} + 15 \cdot 2\sqrt{3} =$

$= \sqrt{3}(12 + 28 - 60 + 30) = (40 - 30)\sqrt{3} =$

$= \boxed{10\sqrt{3}}$

$\sqrt{432} = \sqrt{3 \cdot 144} = \sqrt{3} \cdot \sqrt{144} = 12\sqrt{3}$

$\sqrt{588} = \sqrt{3 \cdot 196} = \sqrt{3} \cdot \sqrt{196} = 14\sqrt{3}$

$\sqrt{1200} = \sqrt{3 \cdot 400} = \sqrt{3} \cdot \sqrt{400} = 20\sqrt{3}$

$\sqrt{12} = \sqrt{3 \cdot 4} = \sqrt{3} \cdot \sqrt{4} = 2\sqrt{3}$

$6{,}25 : 2{,}5 + 0{,}4 : 0{,}8 + 0{,}1(6) : 0{,}1(3) = 2{,}5 + 0{,}5 + 5 = 3 + 5 = 8$

$6{,}25 : 2{,}5 = \frac{6{,}25}{100} : \frac{2{,}5}{10} = \frac{\cancel{6{,}25}^{25}}{\cancel{100}_{10}} \cdot \frac{\cancel{10}^{1}}{\cancel{2{,}5}_{1}} = \frac{25}{10} = \boxed{2{,}5}$

$0{,}4 : 0{,}8 = \frac{4}{10} : \frac{8}{10} = \frac{\cancel{4}^{1}}{\cancel{10}_{10}} \cdot \frac{\cancel{10}^{1}}{\cancel{8}_{2}} = \boxed{\frac{1}{2}} = 0{,}5$

$0{,}1(6) = \frac{6}{9}$

$0{,}1(3) = \frac{13 - 1}{90} = \frac{12}{90}$

$0{,}1(6) : 0{,}1(3) = \frac{6}{9} : \frac{12}{90} = \frac{\cancel{6}^{1}}{\cancel{9}_{3}} \cdot \frac{\cancel{90}^{10}}{\cancel{12}_{2}} = \frac{10}{2} = \boxed{5}$

$2,(4) + 4,(6) + 6,(2) = 2 + 0,(4) + 4 + 0,(6) + 6 + 0,(2) =$

$= 2 + \frac{4}{9} + 4 + \frac{6}{9} + 6 + \frac{2}{9} = 2 + 4 + 6 + \frac{4}{9} + \frac{6}{9} + \frac{2}{9} =$

$= 12 + \frac{12}{9} = 12 + \frac{9+3}{9} = 12 + \frac{9}{9} + \frac{3}{9} = 12 + 1 + \frac{3}{9} = \boxed{13 + \frac{3}{9}}$

$2,4(6) + 4,6(2) + 6,2(4) = 2 + 0,4(6) + 4 + 0,6(2) + 6 + 0,2(4) =$

$= 2 + \frac{46-4}{90} + 4 + \frac{62-6}{90} + 6 + \frac{24-2}{90} =$

$= 2 + 4 + 6 + \frac{42}{90} + \frac{56}{90} + \frac{22}{90} = 12 + \frac{42+56+22}{90} =$

$= 12 + \frac{98+22}{90} = 12 + \frac{120}{90} = 12 + \frac{12}{9} = 12 + \frac{9+3}{9} = 12 + \frac{9}{9} + \frac{3}{9} =$

$= 12 + 1 + \frac{3}{9} = \boxed{13 + \frac{3}{9}}$

$0,(2) = \frac{2}{9}$

$0,(12) = \frac{12}{99}$

$0,(146) = \frac{146}{999}$

$0,1(2) = \frac{12-1}{90} = \frac{11}{90}$

$0,23(34) = \frac{2334-23}{9900} = \frac{2311}{9900}$

$1,(2) = 1 + 0,(2) = 1 + \frac{2}{9} = \frac{9}{9} + \frac{2}{9} = \frac{11}{9}$

Aplicații

$$\left[\sqrt{6561}\cdot\frac{-3,4(1)}{-27,9}-\left(\frac{4}{3}\right)^{-2}\cdot 0,(4)\right]\cdot\left(-\frac{5}{\sqrt{7}}\right)^{2}\cdot\frac{1}{10}=\left(81\cdot\frac{\frac{307}{90}}{\frac{279}{10}}-\left(\frac{3}{4}\right)^{2}\cdot\frac{1}{9}\right)\cdot\frac{6^{2}}{7}\cdot\frac{1}{10}$$

$$=\left(\cancel{81}\cdot\frac{307}{\cancel{90}}\cdot\frac{\cancel{10}}{\cancel{279}}-\frac{\cancel{9}}{\cancel{16}}\cdot\frac{\cancel{4}}{\cancel{9}}\right)\frac{36}{70}=\left(\frac{307}{31}-\frac{1}{4}\right)\cdot\frac{36}{70}=\frac{1228-31}{31\cdot\cancel{4}}\cdot\frac{\cancel{36}}{70}=$$

$$=\frac{1197\cdot 9}{31\cdot 70}$$

$\sqrt{65'61,00'00}\,\big|\,81$
64
$=161$
161
$=:=$

$161\cdot 1=161$

$\Rightarrow\sqrt{6561}=81$

$3,4(1)=3+0,4(1)=3+\frac{41-4}{90}=3+\frac{37}{90}$

$=\frac{270+37}{90}=\frac{307}{90}$

$(a)^{-2}=\frac{1}{a^{2}}$

MODULUL UNUI NR REAL

$$|a|=\begin{cases}+a, & \text{dacă } a>0\\ 0, & \text{dacă } a=0\\ -a, & \text{dacă } a<0\end{cases}$$

Ex:

$|2|=2\geq 0$

$|-2|=-(-2)=2\geq 0$

$|0|=0\geq 0$

$$|E|=\begin{cases}+E, & E\geq 0\\ -E, & E<0\end{cases}$$

Ex:

$$|\underbrace{2x-3}_{E}|=\begin{cases}+(2x-3), & 2x-3\geq 0\\ -(2x-3), & 2x-3<0\end{cases}$$

$$|a| = \begin{cases} +a, & \text{dacă } a \geq 0 \\ -a, & \text{dacă } a < 0 \end{cases}$$

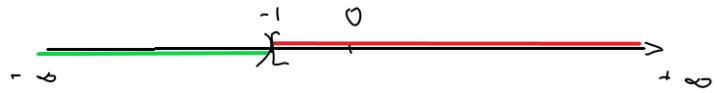
Ex: $|3| = 3$

$|-3| = 3$

$$|E| = \begin{cases} +E, & \text{dacă } E \geq 0 \\ -E, & \text{dacă } E < 0 \end{cases}$$

Ex:

$$|x+1| = \begin{cases} +(x+1), & \text{dacă } x+1 \geq 0 \Leftrightarrow x \geq -1 \Leftrightarrow x \in [-1, +\infty) \\ -(x+1), & \text{dacă } x+1 < 0 \Leftrightarrow x < -1 \Leftrightarrow x \in (-\infty, -1) \end{cases}$$



EXERCIȚII CU MODULE

$|2x-3| + x = 2$

EXPLICITĂM MODULUL $|2x-3|$

$$|2x-3| = \begin{cases} +(2x-3), & \text{dacă } 2x-3 \geq 0 \\ -(2x-3), & \text{dacă } 2x-3 < 0 \end{cases}$$

$$= \begin{cases} +(2x-3), & \text{dacă } 2x \geq 3 \\ -(2x-3), & \text{dacă } 2x < 3 \end{cases} =$$

$$= \begin{cases} +(2x-3), & \text{dacă } x \geq \frac{3}{2} \\ -(2x-3), & \text{dacă } x < \frac{3}{2} \end{cases} =$$

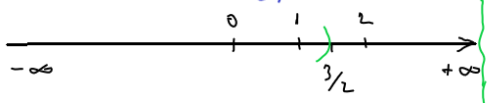
$$= \begin{cases} +(2x-3), & \text{dacă } x \in [\frac{3}{2}, +\infty) \\ -(2x-3), & \text{dacă } x \in (-\infty, \frac{3}{2}) \end{cases}$$

1) Dacă $x \in (-\infty, \frac{3}{2})$ atunci ecuația devine

$-(2x-3) + x = 2 \Leftrightarrow -2x+3+x = 2$

$\Leftrightarrow -x+3 = 2 \Leftrightarrow -2+3 = x \Leftrightarrow$

$\Leftrightarrow x = 1 \in (-\infty, \frac{3}{2})$



$x = 1$ este soluție

Dacă $x \in [\frac{3}{2}, +\infty)$ atunci ecuația devine

$+(2x-3)+x=2 \Leftrightarrow 2x-3+x=2 \Leftrightarrow 3x=2+3$

$\Leftrightarrow 3x=5 \Leftrightarrow x=\frac{5}{3} \in [\frac{3}{2}, +\infty)$, $x=\frac{5}{3}$ este soluție.

verificare

$\frac{3}{2} < \frac{5}{3} \mid \cdot 6$

$\frac{3 \cdot 6}{2} < \frac{5 \cdot 6}{3}$

$9 < 10$ (A)

$|2 \cdot \frac{5}{3} - 3| + \frac{5}{3} = \frac{10}{3} - 3 + \frac{5}{3} =$

$= \frac{10}{3} - \frac{9}{3} + \frac{5}{3} = \frac{1}{3} + \frac{5}{3} = \frac{6}{3} = 2$ (A)

$S = \{1; \frac{5}{3}\}$

Aplicatii cu module

Sa se determine numerele reale care verifica expresiile:

1) $|a| = 8$
2) $|-y| = 2$
3) $|-x-4| = 5$
4) $|x| = -8$
5) $|a+3| = 1$
6) $|3-6y| = 12$
7) $|\frac{x}{2} + \frac{3x}{4} - \frac{5x}{6}| = \frac{15}{2}$
8) $|(x-2)(4-y)| = 0$

1) $a = \pm 8$, $|8| = 8$, $|-8| = 8$

2) $-y = \pm 2 \Leftrightarrow -y = 2$ sau $-y = -2 \Leftrightarrow$

$\Leftrightarrow y = -2$ sau $y = 2$

3) $|-x-4| = 5 \Leftrightarrow -x-4 = \pm 5 \Leftrightarrow$

$\Leftrightarrow -x-4 = 5$ sau $-x-4 = -5$

$\Leftrightarrow -5-4 = x$ sau $5-4 = x$

$\Leftrightarrow x = -9$ sau $x = 1$

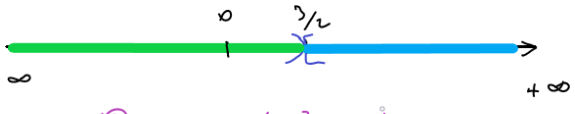
Aplicații EXPLICITĂM MODULUL

$|2x-3| + x = 1$

$|2x-3| = \begin{cases} +(2x-3), & 2x-3 \geq 0 \\ -(2x-3), & 2x-3 < 0 \end{cases} =$

$= \begin{cases} 2x-3, & 2x \geq 3 \Leftrightarrow x \geq \frac{3}{2} \\ -2x+3, & 2x < 3 \Leftrightarrow x < \frac{3}{2} \end{cases}$

$= \begin{cases} 2x-3, & x \in [\frac{3}{2}, +\infty) \\ -2x+3, & x \in (-\infty, \frac{3}{2}) \end{cases}$



1) $x \in (-\infty, \frac{3}{2})$

$-2x+3+x = 1$

$3-1 = 2x-x$

$2 = x \notin (-\infty, \frac{3}{2})$

2) $2x-3+x = 1$

$3x = 3+1$

$3x = 4$

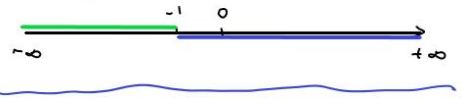
$x = \frac{4}{3} > \frac{3}{2} \quad | \cdot 6$

$\Rightarrow \frac{24}{3} > \frac{18}{2} \Rightarrow 8 > 9$ (F) $\Rightarrow x = \frac{4}{3} \notin [\frac{3}{2}, +\infty)$

$|x+1| - x = 1$

$|x+1| = \begin{cases} +(x+1), & \text{dacă } x+1 \geq 0 \\ -(x+1), & \text{dacă } x+1 < 0 \end{cases} =$

$= \begin{cases} x+1, & x \geq -1 \Leftrightarrow x \in [-1, +\infty) \\ -x-1, & x < -1 \Leftrightarrow x \in (-\infty, -1) \end{cases}$



1) $x \in (-\infty, -1)$ atunci

$-x-1-x = 1$

$-1-1 = x+x \Rightarrow$

$\Rightarrow 2x = -2$

$\Rightarrow x = -1 \notin (-\infty, -1)$

2) $x \in [-1, +\infty)$ atunci

$x+1-x = 1 \Leftrightarrow 1 = 1$ (A), $\forall x \in [-1, +\infty)$

$\Rightarrow S = [-1, +\infty)$

TEMA E5/pag 15

E6, E4/pag 7

Aplicatii cu module

1) $|3x-2| = 5 \Leftrightarrow 3x-2 = 5$ sau $3x-2 = -5 \Leftrightarrow 3x = 7$ sau $3x = -3$ $\Leftrightarrow x = \frac{7}{3}$ sau $x = -1$

2) $|-2-x| = -3 \Leftrightarrow x \in \emptyset$ ($\nexists$ solutie)

3) $\left|\frac{2x}{2} + \frac{3x}{4} - \frac{4}{3}\right| = \frac{1}{2} \Leftrightarrow \left|\frac{12x}{4} + \frac{3x}{4} - \frac{16}{4}\right| = \frac{1}{2} \Leftrightarrow \left|\frac{15x-16}{4}\right| = \frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \frac{15x-16}{4} = \frac{1}{2} \Big| \cdot 4$ sau $\frac{15x-16}{4} = -\frac{1}{2} \Big| \cdot 4 \Leftrightarrow 15x-16 = 2$ sau $15x-16 = -2$

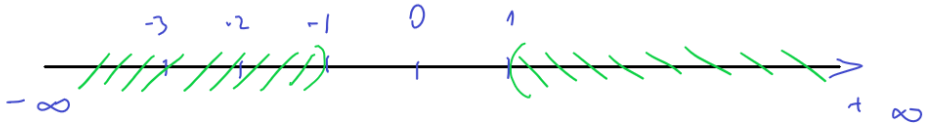
$\Leftrightarrow 15x = 18$ sau $15x = 14 \Leftrightarrow x = \frac{18}{15} = \frac{6}{5}$ sau $x = \frac{14}{15}$

4) $|3x^2-2x-1| = 0$

5) $|2x| < 2$

6) $|2x+1| \leq 0$

Ex. $|x| > 1 \Leftrightarrow x \in (-\infty, -1) \cup (1, +\infty)$



$|2| = 2 > 1$ (A)

$|-2| = 2 > 1$ (A)

$|-3| = 3 > 1$ (A)

b) $|2x+1| \leq 0 \Rightarrow 2x+1 = 0 \Leftrightarrow 2x = -1 \mid :2 \Leftrightarrow x = -\frac{1}{2}$

PROPRIETĂȚI

$|a| \geq 0$

$|a| = 0 \Leftrightarrow a = 0$

$|a \cdot b| = |a| \cdot |b|$

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$|a+b| \leq |a| + |b|$

$|x| \leq k,\ k>0 \Leftrightarrow x \in [-k, +k]$

$|x| > k,\ k>0 \Leftrightarrow x \in (-\infty, -k) \cup (+k, +\infty)$

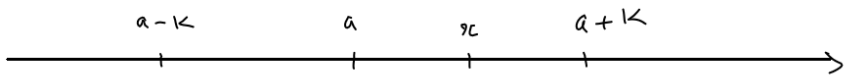
$|x| = k,\ k>0 \Leftrightarrow x = +k$ sau $x = -k$

Aproximari prin lipsa. Aproximari prin adaos.

Numarul a aproximeaza prin lipsa numarul x cu eroare mai mica de k daca $a \leq x \leq a+k$.

Numarul a aproximeaza prin adaos numarul x cu eroare mai mica de k daca $a-k \leq x \leq a$.

Numarul a aproximeaza numarul x cu eroare mai mica de k daca $a-k \leq x \leq a+k$ (sau $|a-x| \leq k$) .



Ex: Fie $x = x_0, x_1 x_2 x_3 \ldots$ un număr real pozitiv ($x > 0$).

Atunci

$x'_m = x_0, x_1 x_2 \ldots x_m$ este aproximare prin lipsă cu eroare mai mică de 10^{-m}

$x''_n = x_0, x_1 x_2 \dots x_n + \frac{1}{10^n}$ aproximare prin adaos cu eroare mai mică de 10^{-n}.

Dacă $x < 0$ atunci

$x'_n = x_0, x_1 x_2 \dots x_n - \frac{1}{10^n}$ este aproximare prin lipsă

$x''_n = x_0, x_1 x_2 \dots x_n$ este aproximare prin adaos.

Aplicatii

Sa se scrie aproximarile zecimale prin lipsa cu eroare de 10^{-2}, 10^{-3} si 10^{-4} pentru numerele

a) $2,45672137 \Rightarrow x'_2 = 2,45$, $x'_3 = 2,456$, $x'_4 = 2,4567$

b) $0,12031671 \Rightarrow x'_2 = 0,12$, $x'_3 = 0,120$, $x'_4 = 0,1203$

c) $\frac{2}{3} = 0,66666\dots \Rightarrow x'_2 = 0,66$, $x'_3 = 0,666$, $x'_4 = 0,6666$

d) $\sqrt{2} = 1,414213562\dots \Rightarrow x'_2 = 1,41$, $x'_3 = 1,414$, $x'_4 = 1,4142$

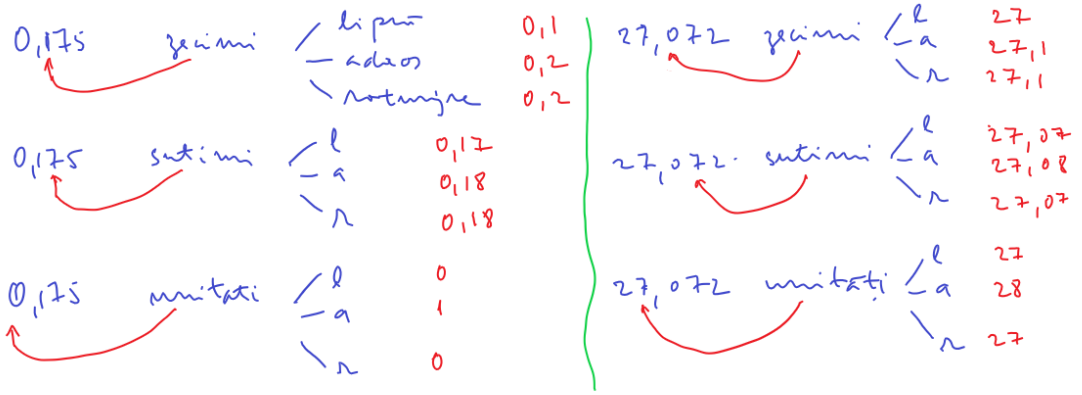
Sa se scrie aproximarile zecimale prin adaos cu eroare 10^{-2}, 10^{-3} si 10^{-4} pentru numerele de mai sus

a) $x''_2 = 2,46$, $x''_3 = 2,457$, $x''_4 = 2,4568$

b) $x''_2 = 0,13$, $x''_3 = 0,121$, $x''_4 = 0,1204$

c) $x''_2 = 0,67$, $x''_3 = 0,667$, $x''_4 = 0,6667$

d) $x''_2 = 1,42$, $x''_3 = 1,415$, $x''_4 = 1,4143$



$2-3=-1$

$-5+7=+2$

$4-9=-5$

$-3-4=-7$

$+3+4=+7$

$-7+2=-5$

$2\cdot(-3)=-6$

$-5\cdot(+7)=-35$

$+4\cdot(-9)=-36$

$-3\cdot(-4)=+12$

$+3\cdot(+4)=+12$

$-7\cdot(+2)=-14$

$a = 2,56 : 1,6 + 2,5 \left[0,5^2 - 0,07 : 10 + 10 \cdot (13 - 12,6) \right] =$

$= 1,6 + 2,5 \left(0,25 - 0,07 + 10 \cdot 0,4 \right) =$

$= 1,6 + 2,5 \left(0,18 + 4 \right) = 1,6 + 2,5 \cdot 4,18 = 1,6 + 10,45 = 12,05$

$b = \frac{2}{19} \left[0,(7) + 0,12(7) \right] : 0,13(8) =$

$= \frac{2}{19} \left(\frac{7}{9} + \frac{27-2}{90} \right) : \frac{138-13}{900} = \frac{2}{19} \cdot \frac{70+25}{90} : \frac{125}{900} =$

$= \frac{2}{19} \cdot \frac{95}{90} \cdot \frac{900}{125} = \frac{2}{19} \cdot \frac{19 \cdot 5 \cdot 10}{125} = \frac{20}{25} = 0,8$

$\Rightarrow a > b$