Aplicatii la inductia matematica

$$1^2+3^2+5^2+\dots+(2n-1)^2 \overset{?}{=} \frac{n(4n^2-1)}{3}, \quad n \geq 1, \quad n \in \mathbb{N}.$$

Rezolvare:

Pas 1. Verificăm enunțul în primul caz, $n=1$.

$$1^2 \overset{?}{=} \frac{1\cdot(4\cdot 1^2-1)}{3} \Leftrightarrow 1 \overset{?}{=} \frac{1\cdot 3}{3} \Leftrightarrow 1 = 1 \ (A)$$

Putem verifica și pt. $n=2$.

$$1^2+3^2 \overset{?}{=} \frac{2\cdot(4\cdot 2^2-1)}{3} \Leftrightarrow 1+9 \overset{?}{=} \frac{2(4\cdot 4-1)}{3} \Leftrightarrow 10 \overset{?}{=} \frac{2\cdot 15}{3} \Leftrightarrow$$

$$\Leftrightarrow 10 = 10 \ (A)$$

Pas 2. Considerăm adevărat enunțul pentru n

Pas 3. Încercăm să demonstrăm, pe baza presupunerii de la pasul 2, pentru $n+1$

$$\underbrace{1^2+3^2+5^2+\dots+(2n-1)^2}_{\frac{n(4n^2-1)}{3}}+(2(n+1)-1)^2 \overset{?}{=} \frac{(n+1)(4(n+1)^2-1)}{3}$$

$$a^2-b^2=(a-b)(a+b)$$

$$\frac{n(4n^2-1)}{3}+(2(n+1)-1)^2 \overset{?}{=} \frac{(n+1)(4(n+1)^2-1)}{3}$$

$$\frac{n(4n^2-1)}{3}+(2n+1)^2 \overset{?}{=} \frac{(n+1)((2(n+1))^2-1^2)}{3} = \frac{(n+1)(2n+1)(2n+3)}{3}$$

$$\frac{n(4n^2-1)}{3}+(2n+1)^2=\frac{n((2n)^2-1)}{3}+(2n+1)^2=$$

$$=\frac{n(2n-1)(2n+1)}{3}+(2n+1)^2=(2n+1)\left[\frac{n(2n-1)}{3}+\overset{3}{/}(2n+1)\right]=$$

$$=(2n+1)\cdot\frac{2n^2-n+6n+3}{3}=\frac{2n+1}{3}\cdot(2n^2+5n+3)=$$

$$=\frac{(2n+1)}{3}(2n^2+2n+3n+3)=\frac{2n+1}{3}\cdot(2n(n+1)+3(n+1))=$$

$$=\frac{2n+1}{3}\cdot(n+1)(2n+3)\quad c.c.t.d.$$

$$a^2-b^2=(a-b)(a+b)$$

Aplicatie

$$1\cdot2\cdot3+3\cdot4\cdot5+\ldots+(2n-1)\cdot2n\cdot(2n+1)\overset{?}{=}n(n+1)(2n^2+2n-1),\ n\geq1,\ n\in\mathbb{N}$$

Rezolvare

Pas 1. Verificare primul caz, $n=1$

$$1\cdot2\cdot3\overset{?}{=}1\cdot(1+1)(2\cdot1^2+2\cdot1-1)\Leftrightarrow 6\overset{?}{=}1\cdot2\cdot(2+2-1)\Leftrightarrow 6=1\cdot2\cdot3\quad(A)$$

Pas 2. Considerăm adevărat pentru n

Pas 3. Încercăm să demonstrăm pentru $n+1$

$$1\cdot2\cdot3+\ldots+(2n-1)2n(2n+1)+(2(n+1)-1)\cdot2(n+1)\cdot(2(n+1)+1)\overset{?}{=}$$

$$\overset{?}{=}(n+1)(n+1+1)(2(n+1)^2+2(n+1)-1)$$

$$\underbrace{1\cdot2\cdot3+\ldots+(2n-1)2n\cdot(2n+1)}_{n(n+1)(2n^2+2n-1)}+(2(n+1)-1)2(n+1)(2(n+1)+1)\overset{?}{=}$$

$$\overset{?}{=}(n+1)(n+1+1)(2(n+1)^2+2(n+1)-1)$$

$$n(n+1)(2n^2+2n-1)+(2n+1)(2n+2)(2n+3)\overset{?}{=}$$

$$\overset{?}{=}(n+1)(n+2)(2(n^2+2n+1)+2n+1)$$

$$(n^2+n)(2n^2+2n-1)+(4n^2+4n+2n+2)(2n+3)\overset{?}{=}$$

$$\overset{?}{=}(n^2+3n+2)(2n^2+4n+2+2n+1)$$

$$2n^4+2n^3-n^2+2n^3+2n^2-n+8n^3+8n^2+4n^2+4n+12n^2+12n+6n+6\overset{?}{=}$$

$$\overset{?}{=}2n^4+4n^3+2n^2+2n^3+n^2+6n^3+12n^2+6n+6n^2+3n+4n^2+8n+$$

$$+4+4n+2\qquad(A)$$