Vectori in plan

Segmente orientate

O pereche ordonata de puncte (A,B) din plan se numeste segment orientat sau vector legat.
Notatie: $\overline{AB}$

A se numeste ORIGINEA

B " EXTREMITATEA

$|\overline{AB}|$ este lungimea segmentului orientat

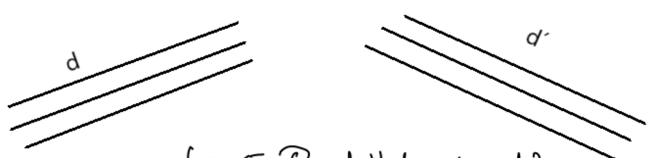
Dreapta AB este DREAPTA SUPORT

Directia dreptei AB este DIRECTIA segmentului orientat.

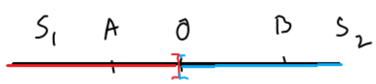
$\overline{AB} = \overline{CD} \iff A = C,\ B = D$

$\overline{AB}, \overline{CD}$ acelasi sens $\iff$ semidreptele (AB si (CD au acelasi sens

Multimea tuturor dreptelor din plan care au aceeasi directie cu dreapta d formeaza directia dreptei d.



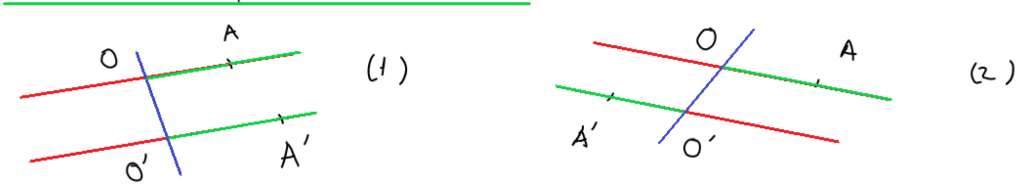
dir(d)= $\{ d_1 \subset \mathcal{P},\ d_1 \parallel d,\ d_1 = d \}$



Deplasarea de la O la A si de la O la B se face in doua sensuri diferite

Semidreptele S1=[OA si S2=[OB determina doua sensuri diferite (opuse)

Sensul a doua semidrepte care au aceeasi directie

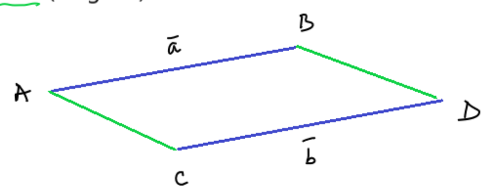


Semidreptele S=[OA si S´=[OA´ au acelasi sens daca sunt incluse in acelasi semiplan (1). In caz contrar au sensuri diferite (2)

Relatia de echipolenta pe multimea segmentelor orientate

Fie P un plan si V multimea segmentelor orientate din plan.

Segmentele orientate $\bar{a}$ si $\bar{b}$ sunt echipolente daca au aceeasi directie, acelasi sens si acelasi modul (lungime).



Notatie: $\bar{a}$ si $\bar{b}$ echipolente $\Leftrightarrow$ $\bar{a} \sim \bar{b}$

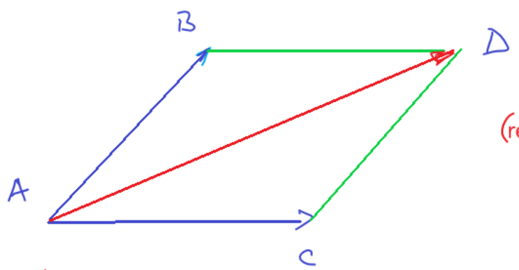
$\bar{a} \sim \bar{b}$ $\Leftrightarrow$ ABDC paralelogram

Se numeste vector liber multimea tuturor segmentelor orientate echipolente cu un segment orientat dat

Notatie: $\vec{v}$

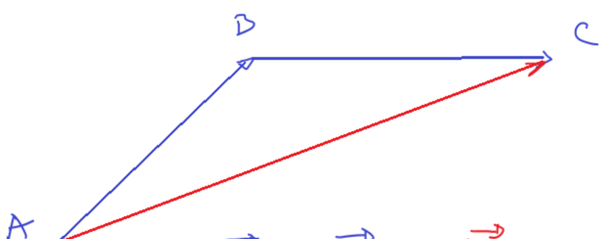
Operatii cu vectori

Adunarea



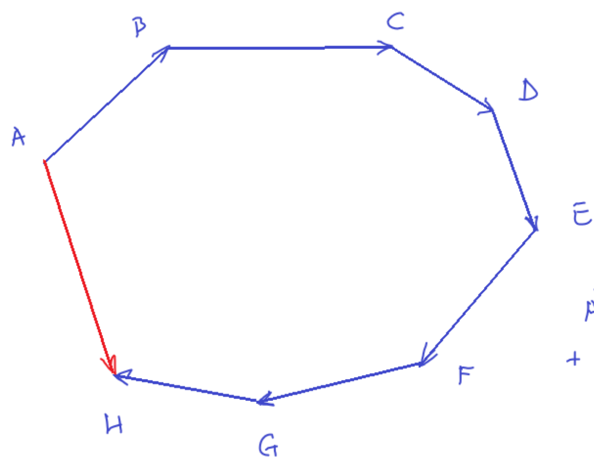
(regula paralelogramului)

$\vec{AB} + \vec{AC} = \vec{AD}$



(regula triunghiului)

$\vec{AB} + \vec{BC} = \vec{AC}$ (rezultanta)

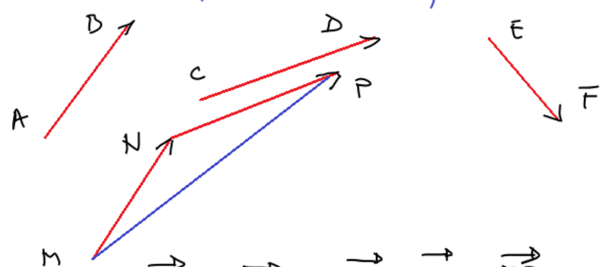


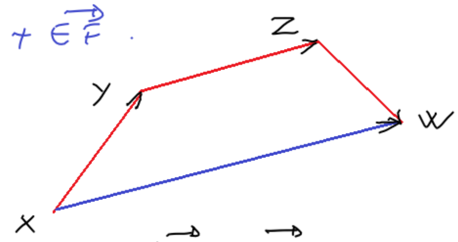
(regula poligonului)

$\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}+$
$+\vec{EF}+\vec{FG}+\vec{GH}=\vec{AH}$

Aplicatii

1) Fie vectorii $\vec{AB}$, $\vec{CD}$, $\vec{EF}$ de mai jos. Efectuați grafic
$\vec{AB}+\vec{CD}$, $\vec{AB}-\vec{CD}$, $\vec{AB}+\vec{CD}+\vec{EF}$.





$\vec{AB}=\vec{MN}$
$\vec{CD}=\vec{NP}$

$\vec{AB}+\vec{CD}=\vec{MN}+\vec{NP}=\vec{MP}$

(regula triunghiului)

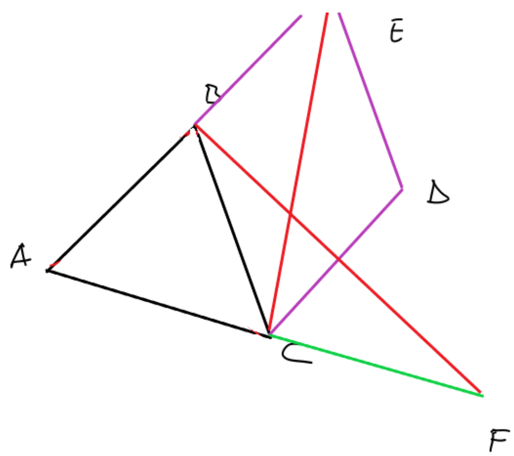
$\vec{AB}=\vec{XY}$
$\vec{CD}=\vec{YZ}$
$\vec{EF}=\vec{ZW}$

$\vec{AB}+\vec{CD}+\vec{EF}=\vec{XY}+\vec{YZ}+\vec{ZW}=$
$=\vec{XW}$

Aplicatii

2) Se considera triunghiul ABC in plan. Sa se exprime sumele:

$\vec{AB}+\vec{BC}=\vec{AC}$
$\vec{AC}+\vec{BA}=\vec{BC}$
$\vec{AB}+\vec{CB}=\vec{CE}$
$\vec{BC}+\vec{AC}=\vec{BF}$



$\vec{AC}+\vec{BA}=\vec{BA}+\vec{AC}=\vec{BC}$
$\vec{AB}+\vec{BC}=\vec{AC}$
$\vec{BC}+\vec{AC}=\vec{BC}+\vec{CF}=\vec{BF}$

Aplicatii

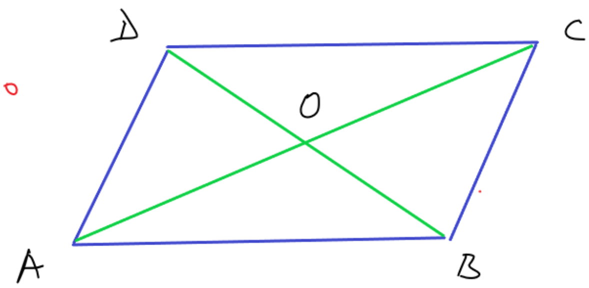
3) Se considera paralelogramul ABCD in plan si se noteaza cu O centrul sau. Sa se exprime sumele

$\vec{AB} + \vec{CO} = \vec{DO}$

$\vec{BC} + \vec{OA} = \vec{BO}$

$\vec{AB} + \vec{OD} = \vec{AO}$

$\vec{AB} + \vec{CO} + \vec{OD} = 0$



$\vec{AB} + \vec{CO} = \vec{DC} + \vec{CO} = \vec{DO}$

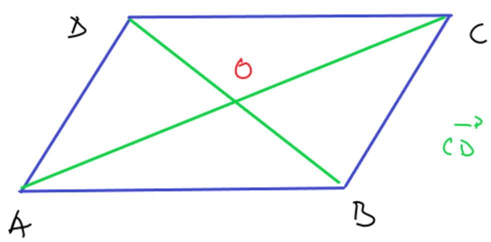
$\vec{BC} + \vec{OA} = \vec{BC} + \vec{CO} = \vec{BO}$

$\vec{AB} + \vec{OD} = \vec{AB} + \vec{BO} = \vec{AO}$

$\vec{AB} + \vec{CO} + \vec{OD} = \vec{DC} + \vec{CO} + \vec{OD} = 0$

Aplicatii

4) Se considera un paralelogram ABCD si fie $\vec{a} = \vec{AC}$, $\vec{b} = \vec{DB}$. Sa se exprime vectorii $\vec{AB}$, $\vec{CB}$, $\vec{CD}$, $\vec{AD}$ in functie de $\vec{a}$ si $\vec{b}$.



$\vec{AD} = \vec{AO} + \vec{OD} = \frac{1}{2}\vec{AC} + \frac{1}{2}\vec{BD} = \frac{1}{2}\vec{a} - \frac{1}{2}\vec{b}$

$\vec{CD} = \vec{CO} + \vec{OD} = \frac{1}{2}\vec{CA} + \frac{1}{2}\vec{BD} = \frac{1}{2}(-\vec{a}) + \frac{1}{2}(-\vec{b}) = -\frac{1}{2}\vec{a} - \frac{1}{2}\vec{b}$

$\vec{AB} = \vec{AD} + \vec{DB} = \vec{AD} + \vec{b} = \frac{1}{2}\vec{a} - \frac{1}{2}\vec{b} + \vec{b} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b}$

$\vec{CB} = \vec{CD} + \vec{DB} = \vec{CD} + \vec{b} = -\frac{1}{2}\vec{a} - \frac{1}{2}\vec{b} + \vec{b} = \frac{1}{2}\vec{b} - \frac{1}{2}\vec{a}$

Aplicatii

5) Fie ABC un triunghi in plan si M, N puncte astfel incat $\vec{MC} = \vec{AB}$ si $\vec{NB} = \vec{BC}$. Ce patrulater formeaza punctele A, B, M, N ?

$\vec{AB} = \vec{MC} \Rightarrow$ ABCM paralelogram $\Rightarrow \vec{AM} = \vec{BC} = \vec{NB} \Rightarrow \vec{AM} = \vec{NB}$

$\Rightarrow$ AMBN paralelogram