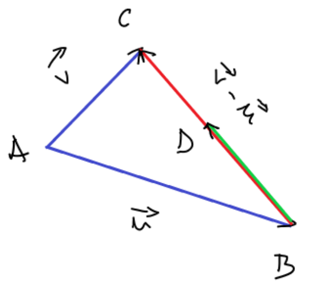
Aplicatie

Se considera vectorii $\vec{u} = \overrightarrow{AB}$ si $\vec{v} = \overrightarrow{AC}$. Sa se scrie $\frac{1}{2}(\vec{u} + \vec{v}) - \vec{u}$.



$$\frac{1}{2}(\vec{u} + \vec{v}) - \vec{u} = \frac{1}{2}\vec{u} + \frac{1}{2}\vec{v} - \vec{u} =$$

$$= \frac{1}{2}\vec{u} - \vec{u} + \frac{1}{2}\vec{v} =$$

$$= -\frac{1}{2}\vec{u} + \frac{1}{2}\vec{v} =$$

$$= \frac{1}{2}(\vec{v} - \vec{u}) =$$

$$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = -\vec{u} + \vec{v} = \vec{v} - \vec{u}$$

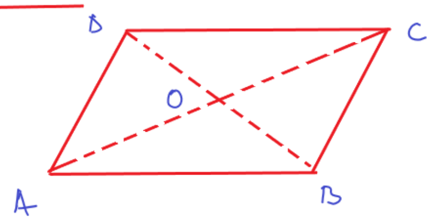
$$= \frac{1}{2}\overrightarrow{BC} = \overrightarrow{BD}$$

unde $DB = DC$.

Aplicatie

Fie paralelogramul ABCD si O mijlocul sau. Sa se exprime sumele $\overrightarrow{AB}+\overrightarrow{OD}$ si $\overrightarrow{AC}+\overrightarrow{CD}$.

Rezolvare



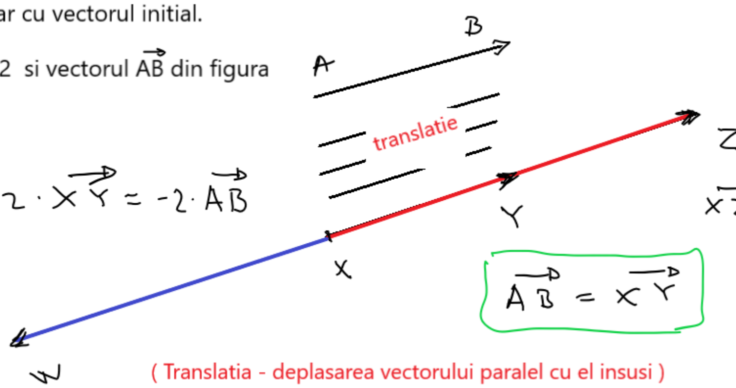
$$\overrightarrow{AB} + \overrightarrow{OD} \overset{\overrightarrow{OD} = \overrightarrow{BO}}{=\!=} \overrightarrow{AB} + \overrightarrow{BO} \underset{\Delta}{\overset{\text{regula}}{=\!=}} \overrightarrow{AC}$$

$$\overrightarrow{AC} + \overrightarrow{CD} \underset{\Delta}{\overset{\text{regula}}{=\!=}} \overrightarrow{AD}$$

Inmultirea unui vector cu un numar real

La inmultirea unui vector cu un numar real se obtine un nou vector de lungime egala cu lungimea vectorului initial inmultita cu numarul real in valoare absoluta. Daca numarul este pozitiv atunci noul vector are aceeasi directie si sens ca si vectorul initial. Daca numarul este negativ atunci noul vector este de aceeasi directie, dar de sens contrar cu vectorul initial.

Ex: a=2, b=-2 si vectorul $\overrightarrow{AB}$ din figura



$\overrightarrow{XW} = -2 \cdot \overrightarrow{XY} = -2 \cdot \overrightarrow{AB}$

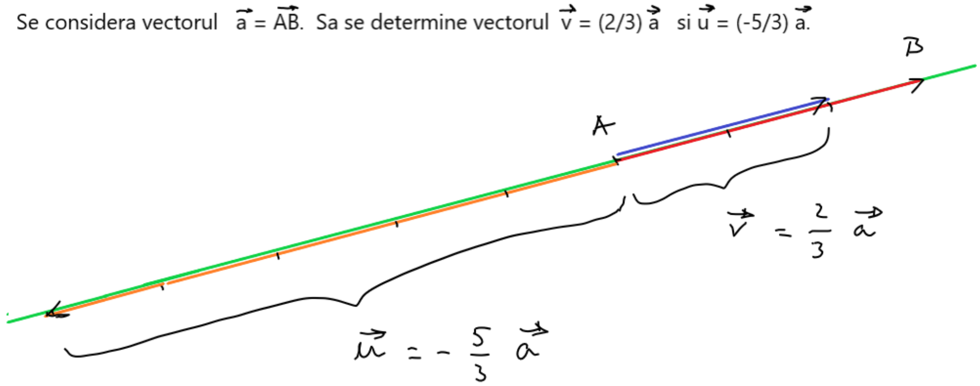
$\overrightarrow{XZ} = 2 \cdot \overrightarrow{XY} = 2 \cdot \overrightarrow{AB}$

$\boxed{\overrightarrow{AB} = \overrightarrow{XY}}$

(Translatia - deplasarea vectorului paralel cu el insusi)

Aplicatie

Se considera vectorul $\vec{a} = \overrightarrow{AB}$. Sa se determine vectorul $\vec{v} = (2/3)\,\vec{a}$ si $\vec{u} = (-5/3)\,\vec{a}$.



$\vec{v} = \frac{2}{3}\vec{a}$

$\vec{u} = -\frac{5}{3}\vec{a}$

Aplicatie

Se considera punctele distincte A si B. Sa se afle $\overrightarrow{AM}$, $\overrightarrow{MA}$, $\overrightarrow{MB}$ daca:

a) M verifica egalitatea $3\overrightarrow{BA} = \overrightarrow{AM}$;

b) $M \in (AB$ si $2\overrightarrow{AM} = 5\overrightarrow{AB}$;

c) M este simetricul lui B in raport cu A.

Rezolvare

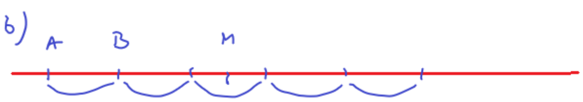
a)

$\overrightarrow{AM} = 3\overrightarrow{BA} = -3\overrightarrow{AB}$

$\overrightarrow{MA} = -3\overrightarrow{BA} = 3\overrightarrow{AB}$

$\overrightarrow{MB} = 4\overrightarrow{AB}$

b)



$\overrightarrow{AM} = 2,5 \cdot \overrightarrow{AB}$

$\overrightarrow{MA} = -2,5 \cdot \overrightarrow{AB}$

$\overrightarrow{MB} = -1,5 \cdot \overrightarrow{AB}$

c)

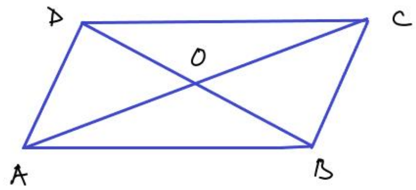
$\overrightarrow{AM} = -\overrightarrow{AB}$

$\overrightarrow{MA} = \overrightarrow{AB}$

$\overrightarrow{MB} = 2 \cdot \overrightarrow{AB}$

Aplicatie

Se considera paralelogramul ABCD cu centrul O. Sa se determine $x \in R$ stiind ca a) $\overrightarrow{AB} = x \cdot \overrightarrow{CD}$.
b) $\overrightarrow{AC} = x \cdot \overrightarrow{OA}$



$\overrightarrow{AB} = \overrightarrow{DC} = -\overrightarrow{CD} = (-1) \cdot \overrightarrow{CD} \Rightarrow$

$\Rightarrow x = -1$

$\overrightarrow{AC} = 2 \cdot \overrightarrow{AO} = -2 \cdot \overrightarrow{OA} \Rightarrow$

$\Rightarrow x = -2$

Aplicatie

Se considera punctele distincte A si B. Sa se afle $\overrightarrow{AM}, \overrightarrow{MA}, \overrightarrow{MB}$ daca $M \in (AB$ si $\overrightarrow{MA} = \frac{2}{3}\overrightarrow{BA}$.

$\overrightarrow{AM} = -\overrightarrow{MA} = -\frac{2}{3}\overrightarrow{BA} = \frac{2}{3}\overrightarrow{AB}$

$\overrightarrow{MA} = -\frac{2}{3}\overrightarrow{AB}$

$\overrightarrow{MB} = \frac{1}{3}\overrightarrow{AB}$

Conditii de coliniaritate

Punctele A, B, C din plan sunt coliniare daca vectorii $\overrightarrow{AB}$ si $\overrightarrow{BC}$ sunt coliniari(si reciproc)

Doi vectori $\vec{a}$ si $\vec{b}$ sunt coliniari daca exista un numar real k astfel incat $\vec{a} = k \cdot \vec{b}$

Punctele A, B, C din plan sunt coliniare daca $|\overrightarrow{AB}| + |\overrightarrow{BC}| = |\overrightarrow{AC}|$ (si reciproc).

Consecinta

Doua drepte sunt paralele daca sunt suporturile a doi vectori coliniari.

Aplicatie

Se considera punctele distincte A si B. Sa se afle $\overrightarrow{AM}$, $\overrightarrow{MA}$, $\overrightarrow{MB}$ daca:

a) M verifica egalitatea $3\overrightarrow{BA} = \overrightarrow{AM}$;

b) $M \in (AB$ si $2\overrightarrow{AM} = 5\overrightarrow{AB}$;

c) M este simetricul lui B in raport cu A.

Rezolvare

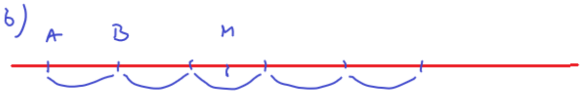
a)

$\overrightarrow{AM} = 3\overrightarrow{BA} = -3\overrightarrow{AB}$

$\overrightarrow{MA} = -3\overrightarrow{BA} = 3\overrightarrow{AB}$

$\overrightarrow{MB} = 4\overrightarrow{AB}$

b)



$\overrightarrow{AM} = 2{,}5 \cdot \overrightarrow{AB}$

$\overrightarrow{MA} = -2{,}5 \cdot \overrightarrow{AB}$

$\overrightarrow{MB} = -1{,}5 \cdot \overrightarrow{AB}$

c)

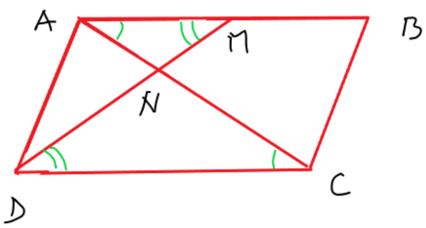
$\overrightarrow{AM} = -\overrightarrow{AB}$

$\overrightarrow{MA} = \overrightarrow{AB}$

$\overrightarrow{MB} = 2 \cdot \overrightarrow{AB}$

Aplicatie

Se considera paralelogramul ABCD si punctele $M\in(AB)$, $N\in(DM)$ astfel incat AM=MB si MD= 3 MN.
Sa se demonstreze ca punctele A, N, C sunt coliniare.



Rezolvare

$$\vec{AN} = \vec{AM} + \vec{MN} \quad | \cdot 2$$
$$\vec{CN} = \vec{CD} + \vec{DN}$$

$$2\vec{AN} + \vec{CN} = 2\vec{AM} + 2\vec{MN} + \vec{CD} + \vec{DN} =$$
$$= 2\underbrace{(\vec{AM} + \vec{MN})}_{\vec{AN}} + \underbrace{\vec{CN}}_{-2\cdot\vec{AN}} = 2\vec{AN} - 2\vec{AN} = 0$$

adica A, N, C coliniare

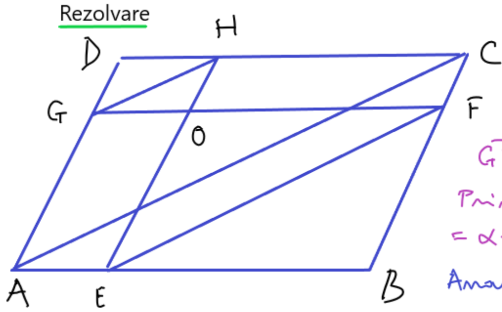
$\triangle AMN \sim \triangle CDN$

$$\frac{DC}{AM} = \frac{CN}{AN} = \frac{DN}{MN} = 2 \Rightarrow \boxed{CN = 2\cdot AN}$$

Aplicatie

Fie ABCD un paralelogram si O un punct interior acestuia. Prin O se duc paralele la laturi GF || AB si EH || AD, $G\in(AD)$, $F\in(BC)$, $E\in(AB)$, $H\in(CD)$.
Sa se arate ca daca $\frac{AE}{AB} + \frac{AG}{AD} = 1$, atunci GH || EF.

Rezolvare



Fie $\alpha = \frac{AE}{AB}$. Rezultă $\frac{AG}{AD} = 1 - \alpha$ și

$\vec{AE} = \alpha\cdot\vec{AB}$, $\vec{AG} = (1-\alpha)\cdot\vec{AD}$.

Putem scrie

$$\vec{GD} = \vec{AD} - \vec{AG} = \vec{AD} - (1-\alpha)\cdot\vec{AD} = \alpha\cdot\vec{AD}$$

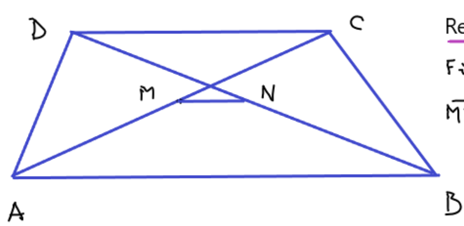
Prin urmare $\vec{GH} = \vec{GD} + \vec{DH} = \alpha\cdot\vec{AD} + \vec{AE} =$
$= \alpha\cdot\vec{AD} + \alpha\cdot\vec{AB} = \alpha\cdot(\vec{AD} + \vec{AB}) = \alpha\cdot\vec{AC} \Rightarrow GH \parallel AC$

Analog $\vec{EF} = \vec{EB} + \vec{BF} = (1-\alpha)\vec{AB} + (1-\alpha)\vec{AD} =$
$= (1-\alpha)\vec{AC} \Rightarrow EF \parallel AC$.

$GH \parallel AC$, $EF \parallel AC \Rightarrow GH \parallel EF$

Aplicatie

Fie ABCD un trapez cu bazele AB si CD, iar M, N mijloacele segmentelor [AC] si [BD]. Sa se arate ca vectorul $\vec{MN}$ este coliniar cu vectorii $\vec{AB}$ si $\vec{CD}$.



Rezolvare

Fie $a = DC$ si $b = AB$, $b > a$.

$\vec{MN} = \vec{MA} + \vec{AB} + \vec{BN}$ (regula poligonului)

$\vec{MN} = \vec{MC} + \vec{CD} + \vec{DN}$

$2\vec{MN} = \cancel{\vec{MA}} + \cancel{\vec{MC}} + \cancel{\vec{BN}} + \cancel{\vec{DN}} + \vec{AB} + \vec{CD}$

$2\vec{MN} = \vec{AB} + \vec{CD} \Rightarrow \vec{MN} = \frac{1}{2}(\vec{AB} - \vec{DC})$

$\vec{MN} = \frac{1}{2}(\alpha\vec{DC} - \vec{DC}) = \frac{\alpha - 1}{2}\cdot\vec{DC} \Rightarrow$

$\Rightarrow \vec{MN}, \vec{DC}$ coliniari $\Rightarrow |\vec{MN}| = \left|\frac{\alpha-1}{2}\right|\cdot|\vec{DC}|$

$\Rightarrow MN = \frac{b-a}{2\cdot \cancel{a}}\cdot\cancel{a} \Rightarrow \boxed{MN = \frac{b-a}{2}}$

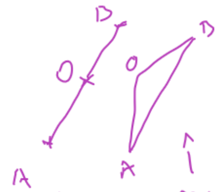
M mijloc $\Rightarrow MA = MC$
N mijloc $\Rightarrow NB = ND$

$AB \parallel CD \Rightarrow \vec{AB} = \alpha\cdot\vec{DC}$
$\Rightarrow |\vec{AB}| = |\alpha|\cdot|\vec{DC}| \Rightarrow$
$\Rightarrow b = \alpha\cdot a \Rightarrow \alpha = \frac{b}{a} > 1$

$$\frac{\alpha - 1}{2} = \frac{\frac{b}{a} - 1}{2} = \frac{b-a}{2a}$$

2) xOy, $A(-1,-1)$, $B(4,4)$; A, O, B coliniare

Pentru a arăta coliniaritatea punctelor A, O, B vom arăta că $\boxed{AO + OB = AB}$

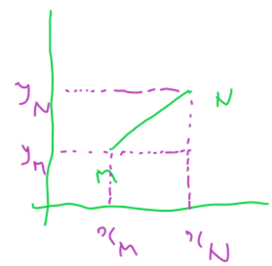


necoliniare $AO + OB > AB$

coliniare

$AO + OB = AB$

$$MN = \sqrt{(x_N - x_M)^2 + (y_N - y_M)^2}$$



$$AO = \sqrt{(x_O - x_A)^2 + (y_O - y_A)^2} =$$

$$= \sqrt{(0-(-1))^2 + (0-(-1))^2} =$$

$$= \sqrt{1+1} = \sqrt{2}$$

$$OB = \sqrt{(x_B - x_O)^2 + (y_B - y_O)^2} = \sqrt{(4-0)^2 + (4-0)^2} = \sqrt{16+16} =$$

$$= \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$$

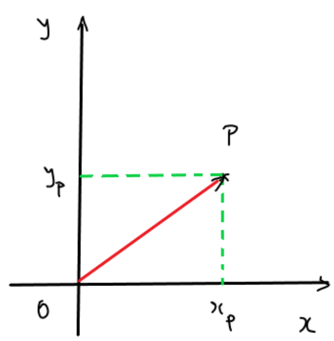
$$AO + OB = \sqrt{2} + 4\sqrt{2} = 5\sqrt{2}$$

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(4-(-1))^2 + (4-(-1))^2} =$$

$$= \sqrt{5^2 + 5^2} = \sqrt{5^2 \cdot 2} = \sqrt{5^2} \cdot \sqrt{2} = 5\sqrt{2}$$

$AO + OB = 5\sqrt{2} = AB \Rightarrow A, O, B$ coliniare

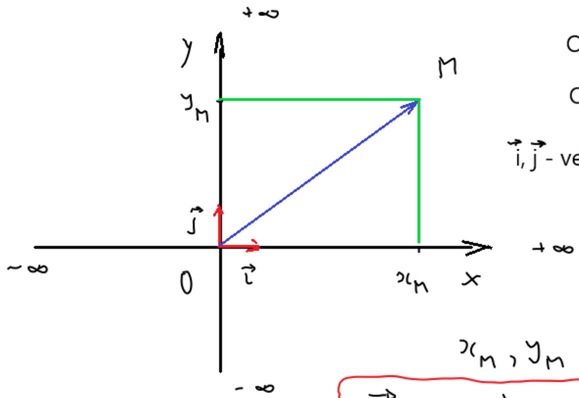
Vectorul de pozitie al unui punct din plan



$\vec{OP} = \vec{r}_P$ se numeste vectorul de pozitie al punctului P

$$\vec{OP} = (x_P, y_P) = \vec{r}_P$$

Reper cartezian in plan. Coordonatele unui vector in plan



Ox - axa absciselor

Oy - axa ordonatelor

$\vec{i}, \vec{j}$ - versori (vectori de lungime 1 pe axe)

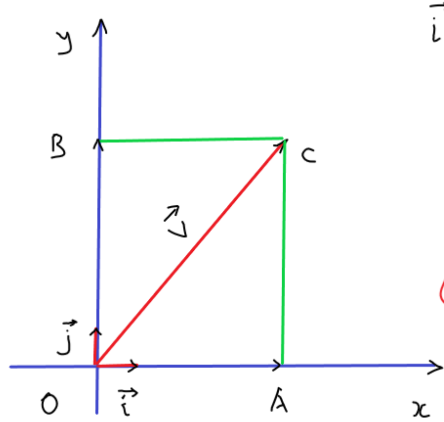
x_M - abscisa lui M

y_M - ordonata lui M

x_M, y_M - coordonatele lui M

$$\boxed{\overrightarrow{OM} = x_M \cdot \vec{i} + y_M \cdot \vec{j}}$$ (scrierea lui $\overrightarrow{OM}$ cu versori)

Descompunerea unui vector in reper cartezian



$\vec{i}, \vec{j}$ - versori (de lungime 1, coliniari cu axele si in sensul acestora)

$\vec{v} = \overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB}$ (regula paralelogramului)

$\overrightarrow{OA} = \alpha \cdot \vec{i}$, $\overrightarrow{OB} = \beta \cdot \vec{j}$

$$\boxed{\vec{v} = \alpha \cdot \vec{i} + \beta \cdot \vec{j}}$$

(descompunerea lui $\vec{v}$ in reper cartezian xOy)

$\overrightarrow{OA} = x_A \cdot \vec{i} + y_A \cdot \vec{j}$ $\qquad$ $\overrightarrow{OA} = (x_A, y_A)$

(exprimarea cu versori) $\qquad$ (exprimarea cu coordonate)

$\overrightarrow{OA} + \overrightarrow{OB} = x_A \vec{i} + y_A \vec{j} +$

$+ x_B \vec{i} + y_B \vec{j} =$

$= (x_A + x_B)\vec{i} + (y_A + y_B)\vec{j}$

$\overrightarrow{OA} = (x_A, y_A)$, $\overrightarrow{OB} = (x_B, y_B)$

$\overrightarrow{OA} + \overrightarrow{OB} = (x_A, y_A) + (x_B, y_B) =$

$= (x_A + x_B, y_A + y_D)$

Aplicatie

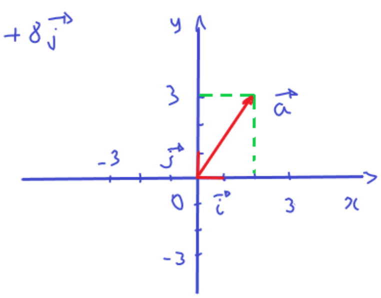
Fie vectorii $\vec{a} = 2\vec{i} + 3\vec{j}$, $\vec{b} = -\vec{i} - 2\vec{j}$, $\vec{c} = 3\vec{i} - 2\vec{j}$.

Sa se calculeze $\vec{v} = 2\vec{a} - 3\vec{b} + 2\vec{c}$.

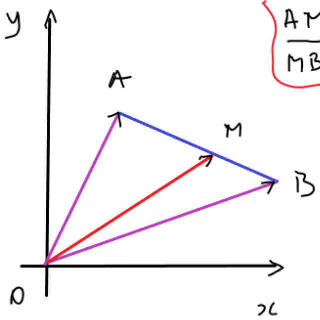
Rezolvare:

$\vec{v} = 2\vec{a} - 3\vec{b} + 2\vec{c} = 2\cdot(2\vec{i} + 3\vec{j}) - 3\cdot(-\vec{i} - 2\vec{j}) + 2\cdot(3\vec{i} - 2\vec{j}) =$

$= 4\vec{i} + 6\vec{j} + 3\vec{i} + 6\vec{j} + 6\vec{i} - 4\vec{j} = 13\vec{i} + 8\vec{j}$



Vectorul de pozitie al punctului care imparte un segment intr-un raport dat

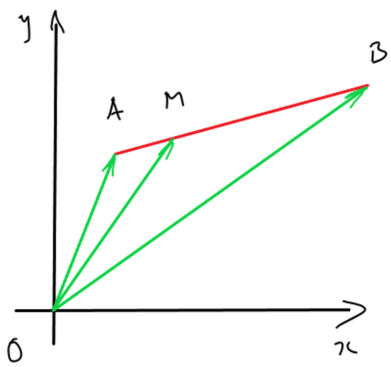


$$\boxed{\frac{AM}{MB} = k}$$

$$\vec{OM} = \frac{1\cdot\vec{OA} + k\cdot\vec{OB}}{1+k}$$

$$\boxed{\vec{OM} = \frac{1}{1+k}\cdot\vec{OA} + \frac{k}{1+k}\cdot\vec{OB}}$$

Coordonatele unui punct care imparte segment in raport dat



$$\vec{AM} = k\cdot\vec{MB}, \quad \vec{OM} = \frac{\vec{OA} + k\cdot\vec{OB}}{1+k} = \frac{1}{1+k}\vec{OA} + \frac{k}{1+k}\cdot\vec{OB}$$

$$\vec{OA} = x_A\cdot\vec{i} + y_A\cdot\vec{j}$$

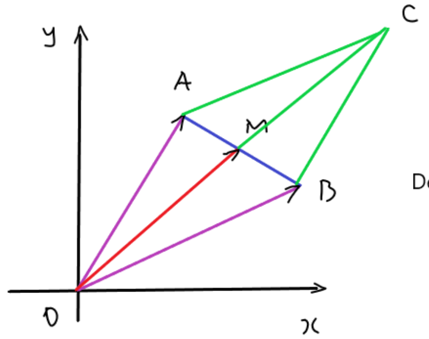
$$\vec{OB} = x_A\cdot\vec{i} + y_A\cdot\vec{j} \Rightarrow \vec{OM} = \frac{1}{1+k}(x_A\vec{i} + y_A\vec{j}) + \frac{k}{1+k}(x_B\vec{i} + y_B\vec{j})$$

$$\vec{OM} = x_M\cdot\vec{i} + y_M\cdot\vec{j}$$

$$\Rightarrow x_M\vec{i} + y_M\vec{j} = \left(\frac{1}{1+k}x_A + \frac{k}{1+k}x_B\right)\vec{i} + \left(\frac{1}{1+k}y_A + \frac{k}{1+k}y_B\right)\vec{j}$$

$$\Rightarrow \begin{cases} x_M = \frac{1}{1+k}\cdot x_A + \frac{k}{1+k}\cdot x_B \\ y_M = \frac{1}{1+k}\cdot y_A + \frac{k}{1+k}\cdot y_B \end{cases}$$

Vectorul de pozitie al mijlocului unui segment



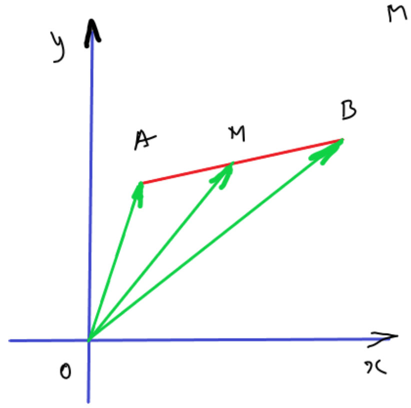
$\vec{OA} + \vec{OB} = \vec{OC}$ (regula paralelogramului)

OC, AB diagonalele paralelogramului OACB $\Rightarrow$

$\Rightarrow OM = \frac{1}{2} OC$.

Deci $\boxed{\vec{OM} = \frac{1}{2}(\vec{OA} + \vec{OB})}$.

Mijlocul unui segment in plan



M mijloc $AB \Rightarrow \vec{OM} = \frac{\vec{OA} + \vec{OB}}{2}$

$\vec{OA} = x_A \cdot \vec{i} + y_A \cdot \vec{j}$

$\vec{OB} = x_B \cdot \vec{i} + y_B \vec{j}$

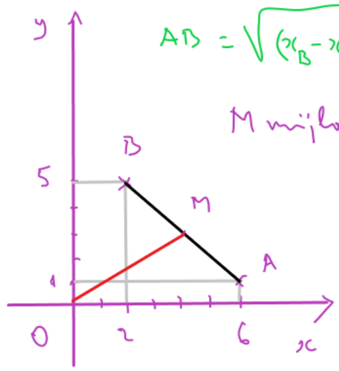
$\Rightarrow \vec{OM} = \frac{1}{2}\left((x_A + x_B)\vec{i} + (y_A + y_B)\vec{j}\right)$

$\vec{OM} = x_M \cdot \vec{i} + y_M \cdot \vec{j}$

$\Rightarrow x_M \vec{i} + y_M \cdot \vec{j} = \frac{1}{2}(x_A + x_B)\vec{i} + \frac{1}{2}(y_A + y_B)\vec{j}$

$$\Rightarrow \begin{cases} x_M = \frac{x_A + x_B}{2} \\ y_M = \frac{y_A + y_B}{2} \end{cases}$$

In reperul xOy se considera A(6,1) si B(2,5). Calculati lungimea segmentului OM, unde M este mijlocul segmentului AB.



$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$, $OM = \sqrt{(x_M - x_O)^2 + (y_M - y_O)^2}$

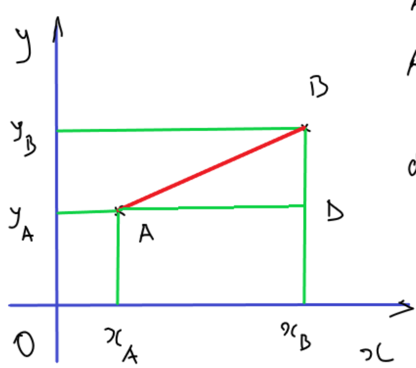
M mijloc AB $\Rightarrow x_M = \frac{x_A + x_B}{2}$, $y_M = \frac{y_A + y_B}{2}$

$x_M = \frac{6+2}{2} = \frac{8}{2} = 4$

$y_M = \frac{1+5}{2} = \frac{6}{2} = 3$

$OM = \sqrt{(4-0)^2 + (3-0)^2} = \sqrt{4^2 + 3^2} = \sqrt{16+9} =$
$= \sqrt{25} = 5$

Distanta dintre doua puncte din plan $A(x_A, y_A)$, $B(x_B, y_B)$

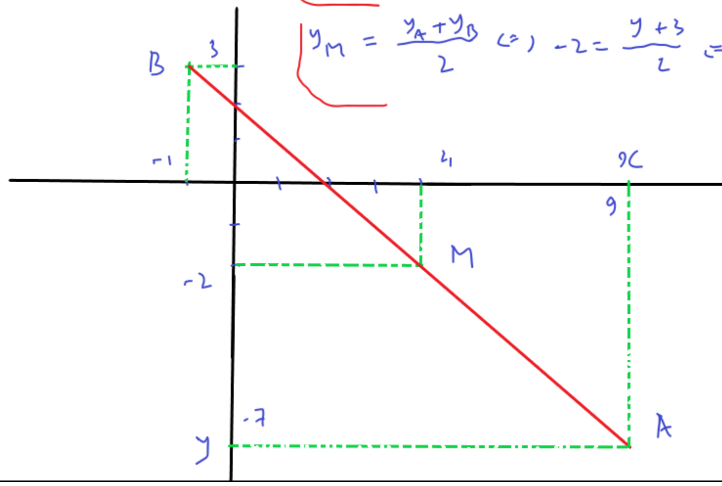


$AB^2 = AD^2 + BD^2$

$AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2$

dist(A,B) = $\boxed{AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}}$

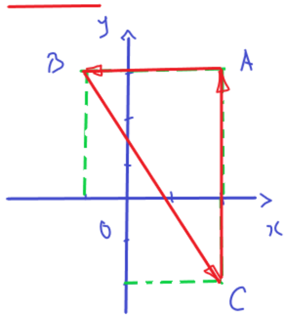
$$\begin{cases} x_M = \frac{x_A + x_B}{2} \Leftrightarrow 4 = \frac{x+(-1)}{2} \Leftrightarrow 8 = x+(-1) \Leftrightarrow x = 9 \\ y_M = \frac{y_A + y_B}{2} \Leftrightarrow -2 = \frac{y+3}{2} \Leftrightarrow -4 = y+3 \Leftrightarrow y = -7 \end{cases}$$



Aplicatie

Fie punctele A(2,3), B(-1,3), C(2,-2). Sa se reprezinte grafic vectorii $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ si sa se afle modulele lor.

Rezolvare



$$|\overrightarrow{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(-1-2)^2 + (3-3)^2} = \sqrt{9+0} = 3$$

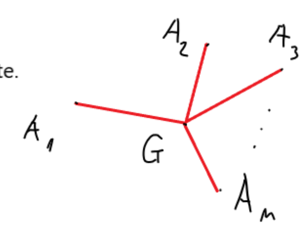
$$|\overrightarrow{BC}| = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} = \sqrt{(2-(-1))^2 + (-2-3)^2} = \sqrt{9+25} = \sqrt{34}$$

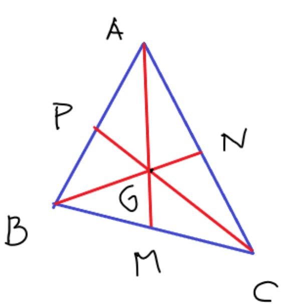
$$|\overrightarrow{CA}| = \sqrt{(x_A - x_C)^2 + (y_A - y_C)^2} = \sqrt{(2-2)^2 + (3-(-2))^2} = \sqrt{0+25} = 5$$

Vectorul de pozitie al centrului de greutate al unui triunghi

Se numeste centrul de greutate al sistemului de puncte A1, A2, ..., An un punct G din plan cu proprietatea ca $\overrightarrow{GA1}+\overrightarrow{GA2}+...+\overrightarrow{GAn} = 0$

Un sistem de puncte A1, A2, ... ,An , n $\in N^*$ din plan admite un singur centru de greutate.



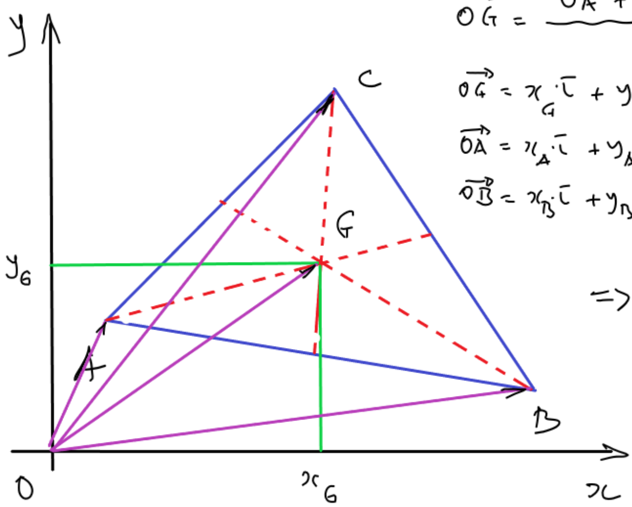


Medianele unui triunghi sunt concurente in centrul de greutate al triunghiului.

$$\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = 0$$

M, N, P mijloacele laturilor BC, AC, AB

Centrul de greutate al unui triunghi



$$\vec{OG} = \frac{\vec{OA} + \vec{OB} + \vec{OC}}{3}$$

$$\vec{OG} = x_G \cdot \vec{i} + y_G \cdot \vec{j}\ , \quad \vec{OC} = x_C \cdot \vec{i} + y_C \cdot \vec{j}$$

$$\vec{OA} = x_A \cdot \vec{i} + y_A \cdot \vec{j}$$

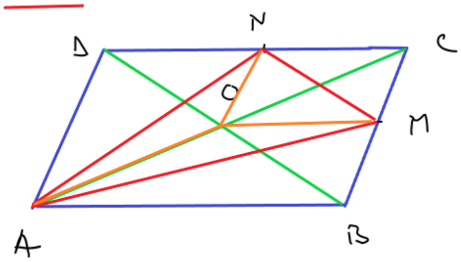
$$\vec{OB} = x_B \cdot \vec{i} + y_B \cdot \vec{j}$$

$$\Rightarrow \begin{cases} x_G = \dfrac{x_A + x_B + x_C}{3} \\ y_G = \dfrac{y_A + y_B + y_C}{3} \end{cases}$$

Aplicatie

Fie paralelogramul ABCD, O centrul sau si $M \in [BC]$, $N \in [CD]$ mijloacele laturilor. Sa se arate ca O este centrul de greutate al triunghiului AMN.

Rezolvare:



$$\vec{OA} = -\vec{AO} = -(\vec{AD} + \vec{AB}) : 2 = -\frac{\vec{AD}}{2} - \frac{\vec{AB}}{2}$$

$$\vec{ON} = \frac{\vec{AD}}{2}$$

$$\vec{OM} = \frac{\vec{AB}}{2}$$

$$\vec{OA} + \vec{ON} + \vec{OM} = -\cancel{\frac{\vec{AD}}{2}} - \cancel{\frac{\vec{AB}}{2}} + \cancel{\frac{\vec{AD}}{2}} + \cancel{\frac{\vec{AB}}{2}} = 0$$

$\Rightarrow$ O este centru de greutate pentru Δ AMN

Aplicatie

Fie punctele A(2,3), B(-3,3), C(1,-2). Sa se determine coordonatele vectorilor $\overrightarrow{AB}$, $\overrightarrow{BC}$.

Rezolvare:

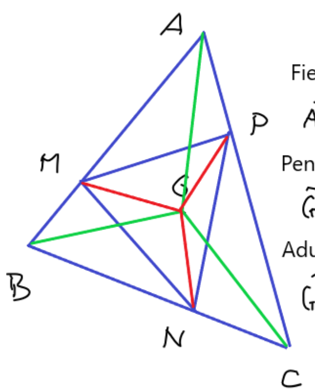
$$\overrightarrow{AB} = (x_B - x_A, y_B - y_A) = (-3-2, 3-3) = (-5, 0) = -5\vec{i}$$

$$\overrightarrow{BC} = (x_C - x_B, y_C - y_B) = (1-(-3), -2-3) = (4, -5) = 4\cdot\vec{i} - 5\cdot\vec{j}$$

$$\overrightarrow{AB} = (x_B - x_A)\cdot\vec{i} + (y_B - y_A)\cdot\vec{j} = (x_B - x_A, y_B - y_A)$$

Aplicatie

Se considera un triunghi ABC si punctele $M \in (AB)$, $N \in (BC)$, $P \in (CA)$ care impart aceste segmente in acelasi raport. Sa se demonstreze ca triunghiurile ABC si MNP au acelasi centru de greutate.



Rezolvare

Fie k raportul in care se impart segmentele. Putem scrie

$$\overrightarrow{AM} = k\cdot\overrightarrow{MB},\ \overrightarrow{BN} = k\cdot\overrightarrow{NC},\ \overrightarrow{CP} = k\cdot\overrightarrow{PA}$$

Pentru centrul de greutate G al triunghiului ABC avem relatiile

$$\overrightarrow{GM} = \frac{\overrightarrow{GA} + k\cdot\overrightarrow{GB}}{1+k},\ \overrightarrow{GN} = \frac{\overrightarrow{GB} + k\cdot\overrightarrow{GC}}{1+k},\ \overrightarrow{GP} = \frac{\overrightarrow{GC} + k\cdot\overrightarrow{GA}}{1+k}$$

Adunand relatiile se obtine

$$\overrightarrow{GM} + \overrightarrow{GN} + \overrightarrow{GP} = \frac{1}{1+k}\left(\overrightarrow{GA} + k\cdot\overrightarrow{GA} + \overrightarrow{GB} + k\cdot\overrightarrow{GB} + \overrightarrow{GC} + k\cdot\overrightarrow{GC}\right) =$$

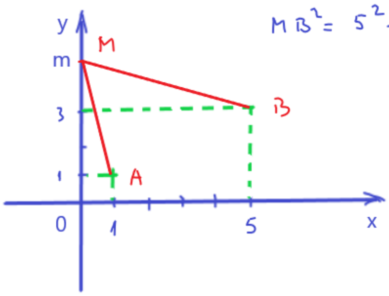
$$= \frac{1}{1+k}\left((1+k)\cdot\overrightarrow{GA} + (1+k)\cdot\overrightarrow{GB} + (1+k)\cdot\overrightarrow{GC}\right) =$$

$= \overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = 0$ (G centru de greutate) $\Rightarrow \overrightarrow{GM} + \overrightarrow{GN} + \overrightarrow{GP} = 0$, adica G centru de greutate si pentru Δ MNP

Aplicatie

Se considera punctele A(1,1), B(5,3). Sa se determine punctele M de pe axele de coordonate pentru care $MA^2 + MB^2 = 36$.

Rezolvare



$MA^2 = 1^2 + (m-1)^2 = 1 + m^2 - 2m + 1 = m^2 - 2m + 2$

$MB^2 = 5^2 + (m-3)^2 = 25 + m^2 - 6m + 9 = m^2 - 6m + 34$

$MA^2 + MB^2 = 36 \iff m^2 - 2m + \cancel{2} + m^2 - 6m + \cancel{34} = \cancel{36}$

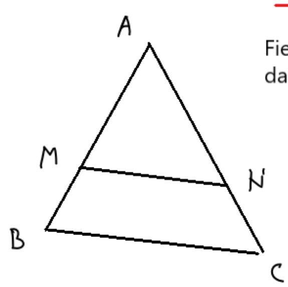
$\iff 2m^2 - 8m = 0 \iff$

$\iff 2m(m-4) = 0 \iff$

$\iff m_1 = 0,\ m_2 = 4$

Similar se procedeaza pe axa orizontala.

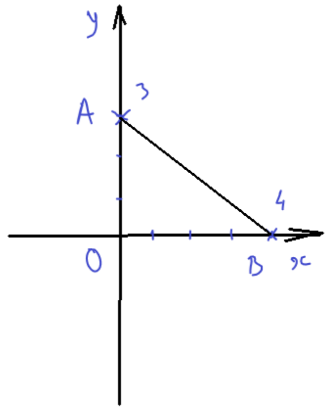
Teorema lui Thales



Fie triunghiul ABC si punctele $M \in AB$, $N \in AC$. Atunci dreapta MN este paralela cu BC daca si numai daca punctele M si N impart segmentele AB si AC in acelasi raport.

$$\boxed{MN \parallel BC \iff \frac{AM}{MB} = \frac{AN}{NC}}$$

In reperul cartezian xOy se considera punctele A(0,3) si B(4,0). Calculati perimetrul triunghiului OAB.



$P_{\Delta OAB} = OA + AB + BO = 3 + 4 + 5 = 12$

$OA = 3$

$OB = 4$

ΔOAB dreptunghic

$AB^2 = OA^2 + OB^2$

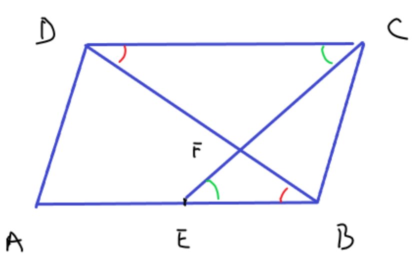
$AB^2 = 3^2 + 4^2 = 9 + 16 = 25$

$AB = \sqrt{25} = 5$

Aplicatie

Punctul E este mijlocul laturii AB a paralelogramului ABCD. Sa se determine raportul in care CE imparte diagonala BD a paralelogramului.

Rezolvare:



$\Delta DFC \sim \Delta BFE \Rightarrow \frac{DF}{BF} = \frac{FC}{FE} = \frac{DC}{BE}$

$\hat{D} = \hat{B}$

$\hat{C} = \hat{E}$

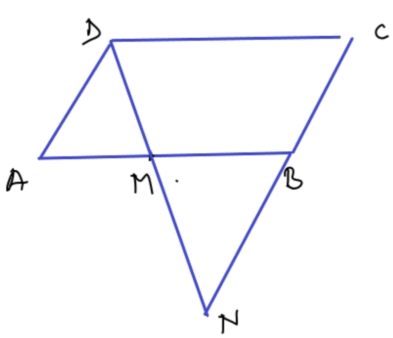
Dar $BE = \frac{AB}{2}$, $DC = AB$.

Atunci $\frac{DF}{BF} = \frac{DC}{\frac{AB}{2}} = \frac{\cancel{AB}}{\frac{\cancel{AB}}{2}} = 2$ c.c.t.d.

Aplicatie

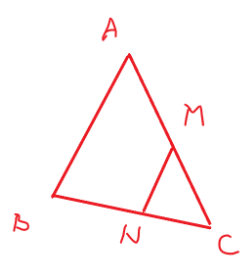
Se considera paralelogramul ABCD si M pe dreapta AB astfel incat A ∈ (MB) si $\frac{MA}{MB} = \frac{2}{3}$. Dreapta MD intersecteaza BC in N. Sa se determine $\frac{BC}{BN}$.

Rezolvare



$AD \parallel BC \Rightarrow \frac{MA}{MB} = \frac{MD}{MN} = \frac{AD}{BN} = \frac{2}{3}$

$AD = BC \Rightarrow \frac{BC}{BN} = \frac{2}{3}$

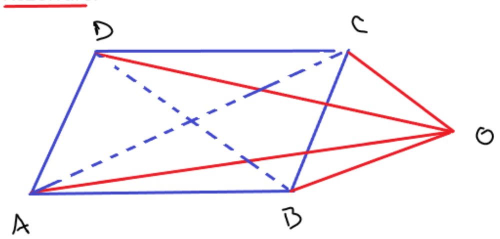


$MN \parallel AB \Leftrightarrow \frac{CM}{CA} = \frac{CN}{CB} = \frac{MN}{AB}$

Aplicatie

Fie ABCD un paralelogram. Sa se demonstreze ca pentru orice punct O din plan are loc egalitatea:
$\overrightarrow{OA} + \overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{OD}$.

Rezolvare:



$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

$\overrightarrow{OC} + \overrightarrow{CD} = \overrightarrow{OD}$

$\overrightarrow{OA} + \overrightarrow{OC} + \cancel{\overrightarrow{AB}} + \cancel{\overrightarrow{CD}} = \overrightarrow{OB} + \overrightarrow{OD}$

$\overrightarrow{OA} + \overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{OD}$ c.c.t.d.

Aplicatii

5) Fie ABC un triunghi in plan si M, N puncte astfel incat $\overrightarrow{MC} = \overrightarrow{AB}$ si $\overrightarrow{NB} = \overrightarrow{BC}$. Ce patrulater formeaza punctele A, B, M, N ?

$\overrightarrow{AB} = \overrightarrow{MC} \Rightarrow ABCM$ paralelogram $\Leftrightarrow \overrightarrow{AM} = \overrightarrow{BC} = \overrightarrow{NB} \Rightarrow \overrightarrow{AM} = \overrightarrow{NB}$

$\Rightarrow$ AMBN paralelogram