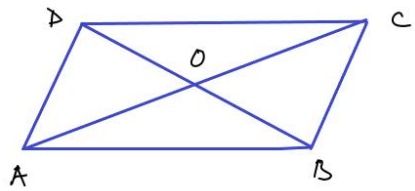
Aplicatie

Se considera paralelogramul ABCD cu centrul O. Sa se determine $x \in R$ stiind ca a) $\vec{AB} = x \cdot \vec{CD}$.

b) $\vec{AC} = x \cdot \vec{OA}$



$\vec{AB} = \vec{DC} = -\vec{CD} = (-1) \cdot \vec{CD} \Rightarrow$

$\Rightarrow x = -1$

$\vec{AC} = 2 \cdot \vec{AO} = -2 \cdot \vec{OA} \Rightarrow$

$\Rightarrow x = -2$

Aplicatie

Se considera punctele distincte A si B. Sa se afle $\overrightarrow{AM}$, $\overrightarrow{MA}$, $\overrightarrow{MB}$ daca:

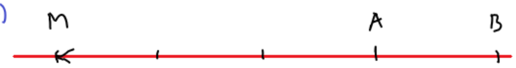
a) M verifica egalitatea $3\overrightarrow{BA} = \overrightarrow{AM}$;

b) M∈(AB si $2\overrightarrow{AM} = 5\overrightarrow{AB}$;

c) M este simetricul lui B in raport cu A.

Rezolvare

a)

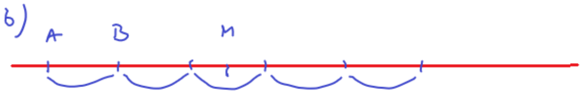


$\overrightarrow{AM} = 3\overrightarrow{BA} = -3\overrightarrow{AB}$

$\overrightarrow{MA} = -3\overrightarrow{BA} = 3\overrightarrow{AB}$

$\overrightarrow{MB} = 4\overrightarrow{AB}$

b)



$\overrightarrow{AM} = 2{,}5\cdot\overrightarrow{AB}$

$\overrightarrow{MA} = -2{,}5\cdot\overrightarrow{AB}$

$\overrightarrow{MB} = -1{,}5\cdot\overrightarrow{AB}$

c)

$\overrightarrow{AM} = -\overrightarrow{AB}$

$\overrightarrow{MA} = \overrightarrow{AB}$

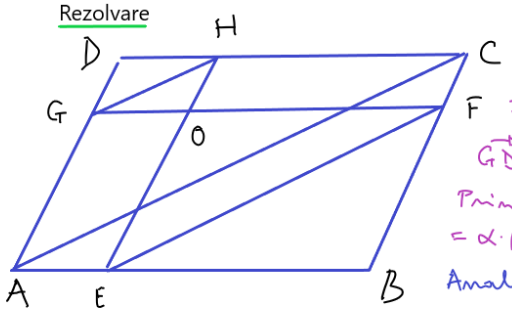
$\overrightarrow{MB} = 2\cdot\overrightarrow{AB}$

Aplicatie

Fie ABCD un paralelogram si O un punct interior acestuia. Prin O se duc paralele la laturi GF || AB si EH || AD, G∈(AD), F∈(BC), E∈(AB), H∈(CD).

Sa se arate ca daca $\frac{AE}{AB} + \frac{AG}{AD} = 1$, atunci GH || EF.

Rezolvare



Fie $\alpha = \frac{AE}{AB}$. Rezultă $\frac{AG}{AD} = 1-\alpha$ și

$\overline{AE} = \alpha\cdot\overline{AB}$, $\overline{AG} = (1-\alpha)\cdot\overline{AD}$.

Putem scrie

$\overrightarrow{GD} = \overrightarrow{AD} - \overrightarrow{AG} = \overrightarrow{AD} - (1-\alpha)\cdot\overrightarrow{AD} = \alpha\cdot\overrightarrow{AD}$

Prin urmare $\overrightarrow{GH} = \overrightarrow{GD} + \overrightarrow{DH} = \alpha\cdot\overrightarrow{AD} + \overrightarrow{AE} =$

$= \alpha\cdot\overrightarrow{AD} + \alpha\cdot\overrightarrow{AB} = \alpha\cdot(\overrightarrow{AD} + \overrightarrow{AB}) = \alpha\cdot\overrightarrow{AC} \Rightarrow$ GH || AC

Analog $\overrightarrow{EF} = \overrightarrow{EB} + \overrightarrow{BF} = (1-\alpha)\overrightarrow{AB} + (1-\alpha)\overrightarrow{AD} =$

$= (1-\alpha)\overrightarrow{AC} \Rightarrow$ EF || AC.

GH || AC, EF || AC ⇒ GH || EF