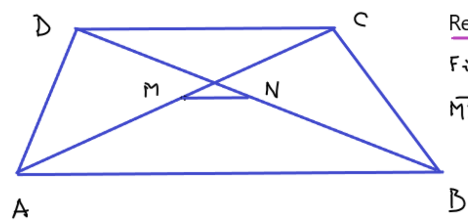
## Aplicatie

Fie ABCD un trapez cu bazele AB si CD, iar M, N mijloacele segmentelor [AC] si [BD]. Sa se arate ca vectorul $\overrightarrow{MN}$ este coliniar cu vectorii $\overrightarrow{AB}$ si $\overrightarrow{CD}$.




### Rezolvare

Fie $a = DC$ si $b = AB$, $b > a$.

$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AB} + \overrightarrow{BN}$ (regula poligonului)

$\overrightarrow{MN} = \overrightarrow{MC} + \overrightarrow{CD} + \overrightarrow{DN}$

$2\overrightarrow{MN} = \cancel{\overrightarrow{MA}} + \cancel{\overrightarrow{MC}} + \cancel{\overrightarrow{BN}} + \cancel{\overrightarrow{DN}} + \overrightarrow{AB} + \overrightarrow{CD}$

M mijloc $\Rightarrow MA = MC$

N mijloc $\Rightarrow NB = ND$

$AB \parallel CD \Rightarrow \overrightarrow{AB} = \alpha \cdot \overrightarrow{DC}$

$\Rightarrow |\overrightarrow{AB}| = |\alpha| \cdot |\overrightarrow{DC}| \Rightarrow$

$\Rightarrow b = \alpha \cdot a \Rightarrow \alpha = \frac{b}{a} > 1$

$\frac{\alpha - 1}{2} = \frac{\frac{b}{a} - 1}{2} = \frac{b-a}{2a}$

$2\overrightarrow{MN} = \overrightarrow{AB} + \overrightarrow{CD} \Rightarrow \overrightarrow{MN} = \frac{1}{2}(\overrightarrow{AB} - \overrightarrow{DC})$

$\overrightarrow{MN} = \frac{1}{2}(\alpha \overrightarrow{DC} - \overrightarrow{DC}) = \frac{\alpha - 1}{2} \cdot \overrightarrow{DC} \Rightarrow$

$\Rightarrow \overrightarrow{MN}, \overrightarrow{DC}$ coliniari $\Rightarrow |\overrightarrow{MN}| = \left|\frac{\alpha-1}{2}\right| \cdot |\overrightarrow{DC}|$

$\Rightarrow MN = \frac{b-a}{2 \cdot \cancel{a}} \cdot \cancel{a} \Rightarrow \boxed{MN = \frac{b-a}{2}}$

## Aplicatie

Se considera punctele distincte A si B. Sa se afle $\overrightarrow{AM}$, $\overrightarrow{MA}$, $\overrightarrow{MB}$ daca $M \in (AB$ si $\overrightarrow{MA} = \frac{2}{3}\overrightarrow{BA}$.

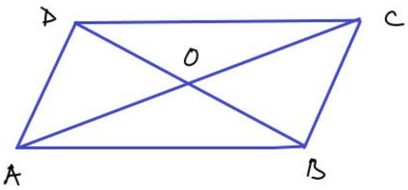

$\overrightarrow{AM} = -\overrightarrow{MA} = -\frac{2}{3}\overrightarrow{BA} = \frac{2}{3}\overrightarrow{AB}$

$\overrightarrow{MA} = -\frac{2}{3}\overrightarrow{AB}$

$\overrightarrow{MB} = \frac{1}{3}\overrightarrow{AB}$

## Aplicatie

Se considera paralelogramul ABCD cu centrul O. Sa se determine x∈R stiind ca a) $\vec{AB} = x\cdot\vec{CD}$.
b) $\vec{AC} = x\cdot\vec{OA}$




$\vec{AB} = \vec{DC} = -\vec{CD} = (-1)\cdot\vec{CD} \Rightarrow$

$\Rightarrow x = -1$

$\vec{AC} = 2\cdot\vec{AO} = -2\cdot\vec{OA} \Rightarrow$

$\Rightarrow x = -2$

## Aplicatie

Se considera punctele distincte A si B. Sa se afle $\vec{AM}$, $\vec{MA}$, $\vec{MB}$ daca:

a) M verifica egalitatea $3\,\vec{BA} = \vec{AM}$;

b) M∈(AB si $2\,\vec{AM} = 5\,\vec{AB}$;

c) M este simetricul lui B in raport cu A.

Rezolvare

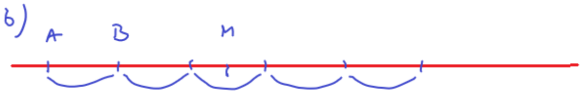
a)


$\vec{AM} = 3\vec{BA} = -3\vec{AB}$

$\vec{MA} = -3\vec{BA} = 3\vec{AB}$

$\vec{MB} = 4\vec{AB}$

b)




$\vec{AM} = 2,5\cdot\vec{AB}$

$\vec{MA} = -2,5\cdot\vec{AB}$

$\vec{MB} = -1,5\cdot\vec{AB}$

c)


$\vec{AM} = -\vec{AB}$

$\vec{MA} = \vec{AB}$

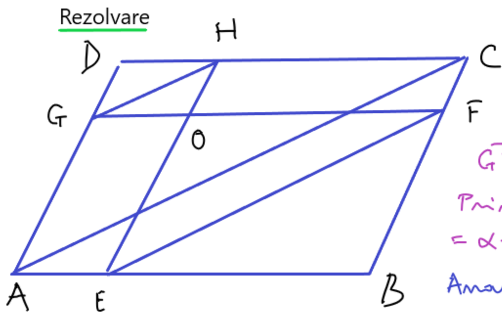
$\vec{MB} = 2\cdot\vec{AB}$

## Aplicatie

Fie ABCD un paralelogram si O un punct interior acestuia. Prin O se duc paralele la laturi GF || AB si EH || AD, G∈(AD), F∈(BC), E∈(AB), H∈(CD).

Sa se arate ca daca $\frac{AE}{AB} + \frac{AG}{AD} = 1$, atunci GH || EF.

Rezolvare




Fie $\alpha = \frac{AE}{AB}$. Rezultă $\frac{AG}{AD} = 1-\alpha$ și

$\vec{AE} = \alpha\cdot\vec{AB}$, $\vec{AG} = (1-\alpha)\cdot\vec{AD}$.

Putem scrie

$\vec{GD} = \vec{AD} - \vec{AG} = \vec{AD} - (1-\alpha)\cdot\vec{AD} = \alpha\cdot\vec{AD}$

Prin urmare $\vec{GH} = \vec{GD} + \vec{DH} = \alpha\cdot\vec{AD} + \vec{AE} =$

$= \alpha\cdot\vec{AD} + \alpha\cdot\vec{AB} = \alpha\cdot(\vec{AD} + \vec{AB}) = \alpha\cdot\vec{AC} \Rightarrow GH \parallel AC$

Analog $\vec{EF} = \vec{EB} + \vec{BF} = (1-\alpha)\vec{AB} + (1-\alpha)\vec{AD} =$

$= (1-\alpha)\vec{AC} \Rightarrow EF \parallel AC$.

$GH \parallel AC,\ EF \parallel AC \Rightarrow GH \parallel EF$