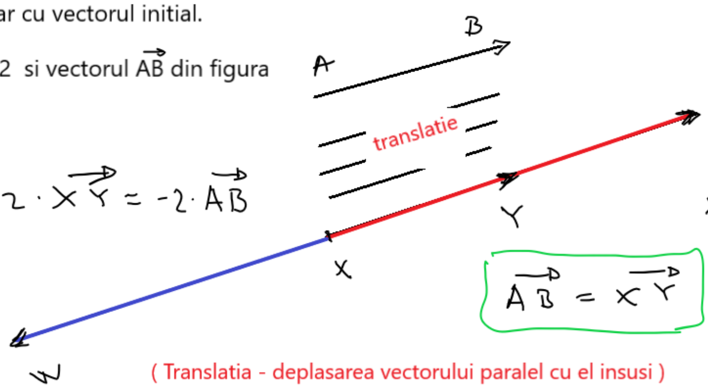
Inmultirea unui vector cu un numar real

La inmultirea unui vector cu un numar real se obtine un nou vector de lungime egala cu lungimea vectorului initial inmultita cu numarul real in valoare absoluta. Daca numarul este pozitiv atunci noul vector are aceeasi directie si sens ca si vectorul initial. Daca numarul este negativ atunci noul vector este de aceeasi directie, dar de sens contrar cu vectorul initial.

Ex: a=2, b=-2 si vectorul $\overrightarrow{AB}$ din figura

$\overrightarrow{XW} = -2 \cdot \overrightarrow{XY} = -2 \cdot \overrightarrow{AB}$



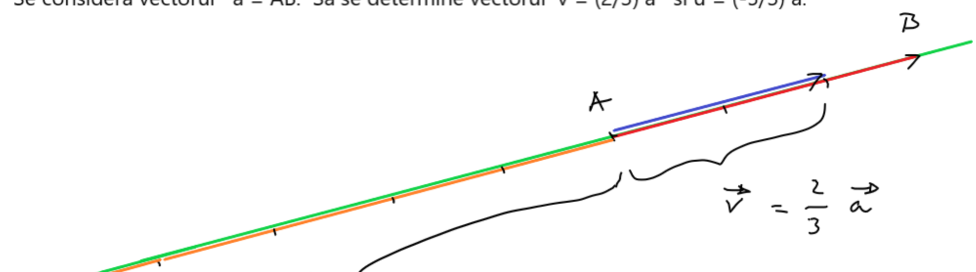
$\boxed{\overrightarrow{AB} = \overrightarrow{XY}}$

$\overrightarrow{XZ} = 2 \cdot \overrightarrow{XY} = 2 \cdot \overrightarrow{AB}$

(Translatia - deplasarea vectorului paralel cu el insusi)

Aplicatie

Se considera vectorul $\vec{a} = \overrightarrow{AB}$. Sa se determine vectorul $\vec{v} = (2/3)\,\vec{a}$ si $\vec{u} = (-5/3)\,\vec{a}$.

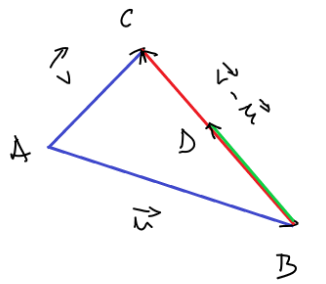


$\vec{v} = \frac{2}{3}\vec{a}$

$\vec{u} = -\frac{5}{3}\vec{a}$

Aplicatie

Se considera vectorii $\vec{u} = \vec{AB}$ si $\vec{v} = \vec{AC}$. Sa se scrie $\frac{1}{2}(\vec{u} + \vec{v}) - \vec{u}$.



$$\frac{1}{2}(\vec{u} + \vec{v}) - \vec{u} = \frac{1}{2}\vec{u} + \frac{1}{2}\vec{v} - \vec{u} =$$
$$= \frac{1}{2}\vec{u} - \vec{u} + \frac{1}{2}\vec{v} =$$
$$= -\frac{1}{2}\vec{u} + \frac{1}{2}\vec{v} =$$
$$= \frac{1}{2}(\vec{v} - \vec{u}) =$$
$$= \frac{1}{2}\vec{BC} = \vec{BD}$$

unde DB = DC.

$$\vec{BC} = \vec{BA} + \vec{AC} = -\vec{u} + \vec{v} = \vec{v} - \vec{u}$$

Conditii de coliniaritate

Punctele A, B, C din plan sunt coliniare daca vectorii $\vec{AB}$ si $\vec{BC}$ sunt coliniari(si reciproc)

Doi vectori $\vec{a}$ si $\vec{b}$ sunt coliniari daca exista un numar real k astfel incat $\vec{a} = k \cdot \vec{b}$

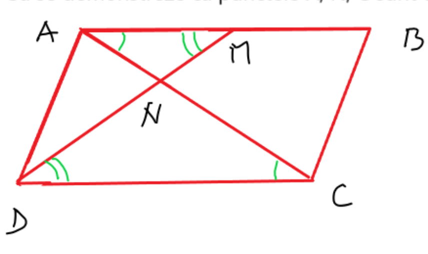
Punctele A, B, C din plan sunt coliniare daca $|\vec{AB}| + |\vec{BC}| = |\vec{AC}|$ (si reciproc).

Consecinta

Doua drepte sunt paralele daca sunt suporturile a doi vectori coliniari.

Aplicatie

Se considera paralelogramul ABCD si punctele M∈(AB), N∈(DM) astfel incat AM=MB si MD= 3 MN.
Sa se demonstreze ca punctele A, N, C sunt coliniare.



$\triangle AMN \sim \triangle CDN$

$$\frac{DC}{AM} = \frac{CN}{AN} = \frac{DN}{MN} = 2 \Rightarrow \boxed{CN = 2 \cdot AN}$$

Rezolvare

$$\vec{AN} = \vec{AM} + \vec{MN} \quad | \cdot 2$$
$$\vec{CN} = \vec{CD} + \vec{DN}$$

$$2\vec{AN} + \vec{CN} = 2\vec{AM} + 2\vec{MN} + \vec{CD} + \vec{DN} =$$
$$= 2(\underbrace{\vec{AM} + \vec{MN}}_{\vec{AN}}) + \underbrace{\vec{CN}}_{-2\cdot\vec{AN}} = 2\vec{AN} - 2\vec{AN} = 0$$

adica A, N, C coliniare