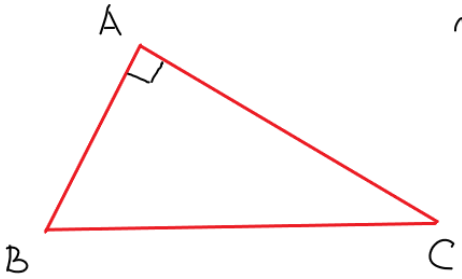
Rezolvarea triunghiului dreptunghic

Prin rezolvarea triunghiului dreptunghic intelegem aflarea tuturor elementelor acestuia cand se *cunosc o parte din ele* (unghiul drept si inca 2 elemente dintre care un element sa fie o latura).



$m(\hat{A}) = 90$

$BC^2 = AB^2 + AC^2$ (teorema lui Pitagora)

$\sin B = \frac{\text{cateta opusa lui B}}{\text{ipotenuza}} = \frac{AC}{BC}$

$\cos B = \frac{\text{cateta alaturata lui B}}{\text{ipotenuza}} = \frac{AB}{BC}$

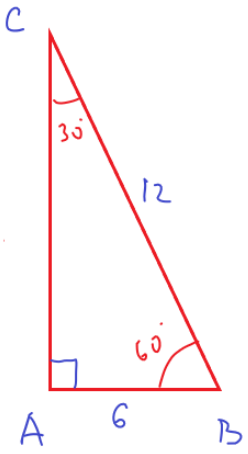
$tg\ B = \frac{\text{cateta opusa lui B}}{\text{cateta alaturata lui B}} = \frac{AC}{AB}$, $ctg\ B = \frac{\text{cateta alaturata lui B}}{\text{cateta opusa lui B}} = \frac{AB}{AC} = \frac{1}{tg\ B}$

Relatiile de mai sus se folosesc la rezolvarea triunghiului dreptunghic.

Aplicatie

Sa se rezolve triunghiul dreptunghic ABC stiind A = 90°, AB = 6, BC = 12.

Rezolvare



$\sin C = \frac{AB}{BC} \Leftrightarrow \sin C = \frac{6}{12} \Leftrightarrow \sin C = \frac{1}{2} \Rightarrow \boxed{C = 30^\circ}$

$B = 90^\circ - C = 90^\circ - 30^\circ \Rightarrow \boxed{B = 60^\circ}$

$BC^2 = AB^2 + AC^2 \Leftrightarrow 12^2 = 6^2 + AC^2 \Leftrightarrow$

$\Leftrightarrow 12^2 - 6^2 = AC^2 \Leftrightarrow (12-6)(12+6) = AC^2$

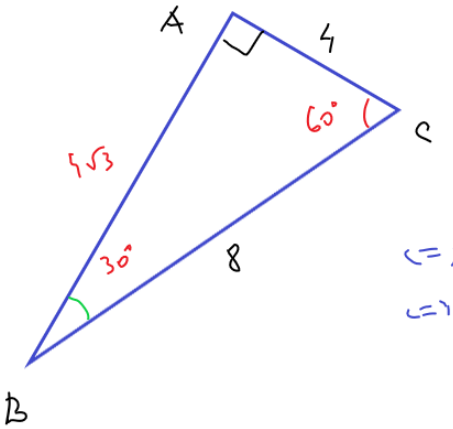
$\Leftrightarrow 6 \cdot 18 = AC^2 \Leftrightarrow 6 \cdot 6 \cdot 3 = AC^2 \Leftrightarrow$

$\Leftrightarrow \boxed{AC = 6\sqrt{3}}$

Aplicatie

Sa se rezolve triunghiul dreptunghic ABC stiind ca A = 90°, AC = 4, BC = 8.

Rezolvare:



$\sin B = \frac{AC}{BC} \Leftrightarrow \sin B = \frac{4}{8} \Leftrightarrow \sin B = \frac{1}{2} \Rightarrow B = 30^\circ$

$B = 30^\circ \Rightarrow C = 90^\circ - 30^\circ = 60^\circ$

$BC^2 = AC^2 + AB^2 \Leftrightarrow 8^2 = 4^2 + AB^2 \Leftrightarrow$

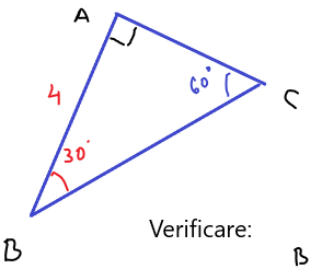
$\Leftrightarrow AB^2 = 8^2 - 4^2 \Leftrightarrow AB = \sqrt{64-16} = \sqrt{48} = \sqrt{16 \cdot 3}$

$\Leftrightarrow AB = 4\sqrt{3}$

Aplicatie

Fie triunghiul dreptunghic ABC cu A=90°, B=30°, AB=4. Sa se rezolve triunghiul.

Rezolvare:



$C = 90^\circ - 30^\circ = 60^\circ$

$tg B = \frac{AC}{AB} \Leftrightarrow tg\, 30^\circ = \frac{AC}{4} \Leftrightarrow \frac{1}{\sqrt{3}} = \frac{AC}{4} \Leftrightarrow AC = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

$\sin B = \frac{AC}{BC} \Leftrightarrow \frac{1}{2} = \frac{\frac{4\sqrt{3}}{3}}{BC} \Leftrightarrow BC = 2 \cdot \frac{4\sqrt{3}}{3} = \frac{8\sqrt{3}}{3}$

Verificare:

$BC^2 = AB^2 + AC^2 \Leftrightarrow \left(\frac{8\sqrt{3}}{3}\right)^2 = 4^2 + \left(\frac{4\sqrt{3}}{3}\right)^2 \Leftrightarrow \frac{64}{3} = 16 + \frac{16}{3} \mid \cdot 3$

(t. Pitagora)

$\Leftrightarrow 64 = 48 + 16 \Leftrightarrow 64 = 64$ (A)