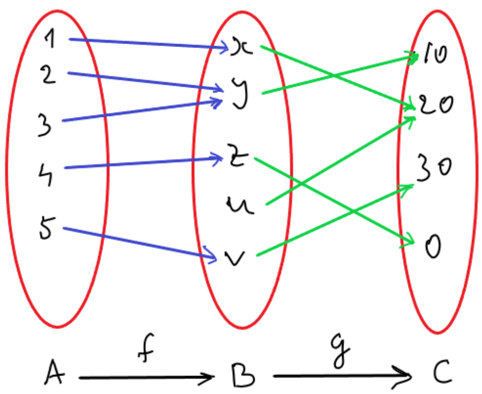
Aplicatie

Se considera functiile f si g definite in diagrama urmatoare :



a) Sa se determine elementele definitorii ale functiei $g \circ f$.

b) Sa se determine $g \circ f(\{2,3\})$ (imaginea)

$g \circ f^{-1}(\{0,20\})$, $g \circ f^{-1}(\{10,30\})$.

(preimaginea)

Rezolvare

a) $A \xrightarrow{g \circ f} C$, A = domeniul , C = codomeniu

b) $g \circ f(2) = g(f(2)) = g(y) = 10$

$g \circ f(3) = g(f(3)) = g(y) = 10$

Deci $g \circ f(\{2,3\}) = \{10\}$.

b) $(g \circ f)^{-1}(\{0,20\}) = \{x \in A, (g \circ f)(x) \in \{0,20\}\} = \{1,4\}$

$(g \circ f)^{-1}(\{10,30\}) = \{x \in A, (g \circ f)(x) \in \{10,30\}\} = \{2,3,5\}$

Aplicatii

$\boxed{f \circ g(x) = f(g(x))}$

Fie functiile f,g: R ⟶ R. Sa se determine $f \circ g$, $g \circ f$, $f \circ f$ și $g \circ g$ dacă

$f(x) = 6x - 5$, $g(x) = \frac{1}{6}x + \frac{5}{6}$. („o" inseamna operatia de compunere a functiilor scrise)

Rezolvare:

$f \circ g(x) = f(g(x)) = f(\underbrace{\tfrac{1}{6}x + \tfrac{5}{6}}_{u}) = f(u) = 6u - 5 = 6\left(\frac{1}{6}x + \frac{5}{6}\right) - 5 = x + 5 - 5 = x$

$g \circ f(x) = g(f(x)) = g(\underbrace{6x-5}_{u}) = g(u) = \frac{1}{6}u + \frac{5}{6} = \frac{1}{6}(6x-5) + \frac{5}{6} = x - \frac{5}{6} + \frac{5}{6} = x$

$f \circ f(x) = f(f(x)) = f(\underbrace{6x-5}_{u}) = f(u) = 6u - 5 = 6(6x-5) - 5 = 36x - 30 - 5 = 36x - 35$

$g \circ g(x) = g(g(x)) = g(\underbrace{\tfrac{1}{6}x + \tfrac{5}{6}}_{u}) = g(u) = \frac{1}{6}u + \frac{5}{6} = \frac{1}{6}\left(\frac{1}{6}x + \frac{5}{6}\right) + \frac{5}{6} = \frac{1}{36}x + \frac{5}{36} + {}^{6/}\frac{5}{6} =$

$= \frac{1}{36}x + \frac{5+30}{36} = \frac{1}{36}x + \frac{35}{36}$

Aplicatie

Sa se determine fof, fog, gof, gog daca $f, g: \mathbb{R} \to \mathbb{R}$, $f(x) = \sqrt{x^2+x+2}$, $g(x) = (x-1)^2$.

Rezolvare:

$$f\circ f(x) = f(\underbrace{f(x)}_{u}) = f(u) = \sqrt{u^2+u+2} = \sqrt{(f(x))^2 + f(x) + 2} =$$

$$= \sqrt{\left(\sqrt{x^2+x+2}\right)^2 + \sqrt{x^2+x+2} + 2} = \sqrt{x^2+x+\sqrt{x^2+x+2} + 4}$$

$$f\circ g(x) = f(\underbrace{g(x)}_{u}) = f(u) = \sqrt{u^2+u+2} = \sqrt{(g(x))^2+g(x)+2} = \sqrt{(x-1)^4+(x-1)^2+2}$$

$$g\circ g(x) = g(\underbrace{g(x)}_{u}) = g(u) = (u-1)^2 = (g(x)-1)^2 = \left[(x-1)^2-1\right]^2$$

$$g\circ f(x) = g(\underbrace{f(x)}_{u}) = g(u) = (u-1)^2 = (f(x)-1)^2 = \left(\sqrt{x^2+x+2} - 1\right)^2 =$$

$$= x^2+x+2-2\sqrt{x^2+x+2} + 1 = x^2+x-2\sqrt{x^2+x+2} + 3$$

Aplicatie

Se considera functiile $f: \mathbb{R} \to \mathbb{R}$, $f(x) = -x+3$, $g: \mathbb{R} \to \mathbb{R}$, $g(x) = \begin{cases} 3x+4, & x \le 1 \\ -x, & x > 1 \end{cases}$.

Sa se determine functiile $f\circ g$ si $g\circ f$.

Rezolvare

$$f\circ g(x) = f(\underbrace{g(x)}_{u}) = f(u) = -u+3 = -g(x)+3 = \begin{cases} -(3x+4)+3, & x \le 1 \\ -(-x)+3, & x > 1 \end{cases} = \begin{cases} -3x-1, & x \le 1 \\ x+3, & x > 1 \end{cases}$$

$$g\circ f(x) = g(\underbrace{f(x)}_{u}) = g(u) = \begin{cases} 3u+4, & u \le 1 \\ -u, & u > 1 \end{cases} = \begin{cases} 3f(x)+4, & f(x) \le 1 \\ -f(x), & f(x) > 1 \end{cases} =$$

$$= \begin{cases} 3(-x+3)+4, & -x+3 \le 1 \\ -(-x+3), & -x+3 > 1 \end{cases} = \begin{cases} -3x+13, & 3-1 \le x \\ x-3, & 3-1 > x \end{cases} = \begin{cases} -3x+13, & x \ge 2 \\ x-3, & x < 2 \end{cases}.$$

Aplicatie

$\boxed{f \circ g(x) \overset{def}{=} f(g(x))}$

Se dau functiile f: R $\longrightarrow$ R, f(x)=3x-5, g:R $\longrightarrow$ R, g(x)=x^2-2. Sa se determine fog si gof.

Rezolvare:

$$f \circ g(x) = f(g(x)) = f(\underbrace{x^2-2}_{u}) = f(u) = 3u - 5 =$$
$$= 3\cdot(x^2-2) - 5 = 3x^2 - 6 - 5 = 3x^2 - 11$$

$$g \circ f(x) = g(f(x)) = g(\underbrace{3x-5}_{u}) = g(u) = u^2 - 2 =$$
$$= (3x-5)^2 - 2 = (3x)^2 - 2\cdot 3x \cdot 5 + 5^2 - 2 = 9x^2 - 30x + 23$$

$$\boxed{(a-b)^2 = a^2 - 2ab + b^2}$$

Aplicatii

Fie functiile f, g : R $\longrightarrow$ R. Sa se determine $f \circ g$, $g \circ f$, $f \circ f$, $g \circ g$ daca $f(x) = 3x - 4$, $g(x) = -x^2 + 1$.

$\boxed{f \circ g(x) = f(g(x))}$

Rezolvare:

$$f \circ g(x) = f(g(x)) = f(\underbrace{-x^2+1}_{u}) = f(u) = 3u - 4 = 3(-x^2+1) - 4 = -3x^2 + 3 - 4 = -3x^2 - 1$$

$$g \circ f(x) = g(f(x)) = g(\underbrace{3x-4}_{u}) = g(u) = -u^2 + 1 = -(3x-4)^2 + 1 = -(9x^2 - 24x + 16) + 1 =$$
$$= -9x^2 + 24x - 16 + 1 = -9x^2 + 24x - 15$$

$$f \circ f(x) = f(f(x)) = f(\underbrace{3x-4}_{u}) = f(u) = 3u - 4 = 3(3x-4) - 4 = 9x - 12 - 4 = 9x - 16$$

$$g \circ g(x) = g(g(x)) = g(\underbrace{-x^2+1}_{u}) = g(u) = -u^2 + 1 = -(-x^2+1)^2 + 1 = -((-x^2)^2 - 2x^2 + 1) + 1 =$$
$$= -(x^4 - 2x^2 + 1) + 1 = -x^4 + 2x^2$$

$$\boxed{\begin{aligned}(a-b)^2 &= a^2 - 2ab + b^2\\ (a+b)^2 &= a^2 + 2ab + b^2\end{aligned}}$$

Aplicatie

Fie functia $f : R \longrightarrow R$, $f(x)=3x+7$. Sa se determine functia $g : R \longrightarrow R$ stiind ca $(g \circ f)(x)=-6x-5, \forall x \in R$.

Rezolvare:

$$g \circ f(x) = g(f(x)) = g(\underbrace{3x+7}_{u}) = g(u) = au+b = a(3x+7)+b = 3ax+7a+b$$

$\forall x \in R$

Fie $g(x) = ax+b$

$$\left.\begin{array}{l}(g \circ f)(x) = -6x-5, \forall x \in R \\ (g \circ f)(x) = 3ax+7a+b, \forall x \in R\end{array}\right\} \Rightarrow -6x-5 = 3ax+7a+b, \forall x \in R$$

$$\Rightarrow 3ax+6x+7a+b+5=0, \forall x \in R \Rightarrow (3a+6)x+7a+b+5=0, \forall x \in R$$

$$\Rightarrow \begin{cases} 3a+6=0 \\ 7a+b+5=0 \end{cases} \Rightarrow \begin{cases} a = \frac{-6}{3} = -2 \\ 7\cdot(-2)+b+5=0 \end{cases} \Rightarrow \begin{cases} a=-2 \\ b=-5+14 \end{cases} \Rightarrow \begin{cases} a=-2 \\ b=9 \end{cases}$$

Deci $g(x) = -2x+9$.

Functia de gradul I

Functia $f: R \longrightarrow R$, $f(x) = ax+b$, $a,b \in R$ se numeste functie afina.

Functia afina f cu $a \neq 0$ se numeste functie de gradul I.

Functia de gradul I cu $b = 0$ se numeste functie liniara. ($f(x) = ax$)

Functia afina f cu $a = 0$ se numeste functie constanta. ($f(x) = b$)

Intersectia graficului functiei de gradul I cu axele.

$f(x) = ax+b$, $a,b \in R$, $a \neq 0$, $x \in R$

$G_f = \{(x,y) \mid y = f(x), x \in \text{Domeniu} = R\}$

$Ox \cap G_f = \{(-\frac{b}{a}, 0)\}$

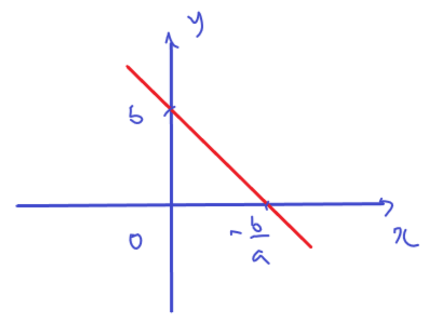
$y=0 \Rightarrow f(x)=0 \Rightarrow ax+b=0 \Rightarrow$

$\Rightarrow ax=-b \Rightarrow x = -\frac{b}{a}$

$Oy \cap G_f = \{(0,b)\}$

$x=0 \Rightarrow y=f(0) \Rightarrow y=a\cdot 0+b$

$\Rightarrow y=b$



Aplicatie

Sa se reprezinte grafic urmatoarea functie afina

$$f:\mathbb{R}\to\mathbb{R},\quad f(x)=\begin{cases}-x+1, & x\le -1\\ 2, & -1<x<1\\ x+1, & x\ge 1\end{cases}$$

Rezolvare:

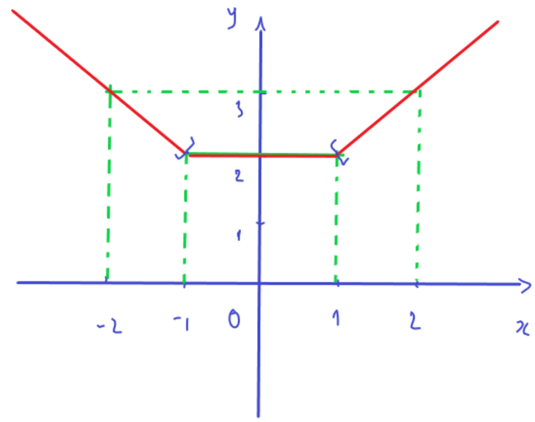
x	$-\infty$	-2	-1	1	2	$+\infty$
$f(x)=-x+1$		3	2]			
$f(x)=2$			(2	2)		
$f(x)=x+1$				[2	3	

$f(-2)=-(-2)+1=2+1=3$

$f(-1)=-(-1)+1=1+1=2$

$f(1)=1+1=2$

$f(2)=2+1=3$



Aplicatie

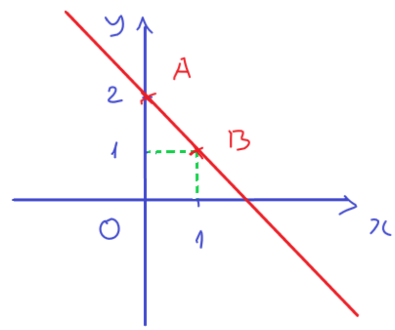
Sa se traseze graficul functiei f:R ⟶ R, f(x)=-x+2 determinand doua puncte arbitrare ale acestuia.

Rezolvare:

x	0	1
$f(x)$	2	1
	A	B

$x=0 \Rightarrow f(0)=-0+2=2 \Rightarrow A(0,2)$

$x=1 \Rightarrow f(1)=-1+2=1 \Rightarrow B(1,1)$



Aplicatie

Fie functia $f: R \longrightarrow R$, $f(x)=(3m-2)x+5$, $m \in R$. Sa se determine $m \in R$ stiind ca $(1,6) \in G_f$.

Rezolvare

$(1,6) \in G_f \Rightarrow f(1) = 6 \Rightarrow (3m-2)\cdot 1 + 5 = 6 \Rightarrow$

x

$\Rightarrow 3m - 2 + 5 = 6 \Rightarrow 3m = 6 - 3 \Rightarrow 3m = 3 \Rightarrow$

$y=f(x)$

$\Rightarrow m = 1$

$G_f = \{(x,y) \mid y = f(x), x \in D\}$

Aplicatie

Sa se determine intersectia cu axele de coordonate a graficului functiei $f(x) = \frac{1}{2}x - 3$, $x \in R$.

Rezolvare:

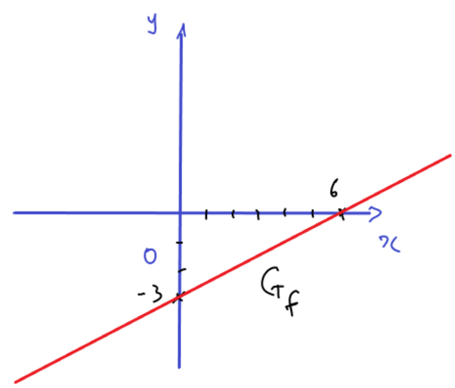
$Oy \cap G_f = \{(0,-3)\}$

$x = 0 \Rightarrow y = f(0) = \frac{1}{2}\cdot 0 - 3 = -3$

$Ox \cap G_f = \{(6,0)\}$

$y = 0 \Rightarrow \frac{1}{2}x - 3 = 0 \mid \cdot 2 \Rightarrow x - 6 = 0$

$\Rightarrow x = 6$



Aplicatie

Sa se determine functia de gradul I $f : \mathbb{R} \longrightarrow \mathbb{R}$, stiind ca $f(1) = 3$ si $f(0) = 2$.

Rezolvare:

f functie de gradul I $\Rightarrow f(x) = ax + b,\ a \neq 0$

$$\begin{cases} f(1) = 3 \\ f(0) = 2 \end{cases} \Rightarrow \begin{cases} a + b = 3 \\ b = 2 \end{cases} \Rightarrow \begin{cases} a = 1 \\ b = 2 \end{cases} \Rightarrow f(x) = x + 2$$