Aplicatie

$$\begin{cases} a_4 + a_8 = 30 \\ 10\cdot a_1 - 4\cdot a_7 = -45 \end{cases} \qquad \begin{aligned} a_1 &= ? \\ r &= ? \end{aligned}$$

$$a_4 = a_1 + (4-1)r = a_1 + 3r$$
$$a_8 = a_1 + (8-1)r = a_1 + 7r$$
$$a_7 = a_1 + (7-1)r = a_1 + 6r$$

$$\begin{cases} a_1 + 3r + a_1 + 7r = 30 \\ 10\cdot a_1 - 4(a_1 + 6r) = -45 \end{cases} \Leftrightarrow \begin{cases} 2a_1 + 10r = 30 \\ 10\cdot a_1 - 4\cdot a_1 - 24r = -45 \end{cases} \Leftrightarrow \begin{cases} 2a_1 + 10r = 30 \quad |:2 \\ 6a_1 - 24r = -45 \end{cases} \Leftrightarrow$$

$$\begin{cases} a_1 + 5r = 15 \\ 6a_1 - 24r = -45 \end{cases} \Leftrightarrow \begin{cases} a_1 = 15 - 5r \\ 6(15 - 5r) - 24r = -45 \end{cases} \Leftrightarrow \begin{cases} a_1 = 15 - 5r \\ 90 - 30r - 24r = -45 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a_1 = 15 - 5r \\ 135 = 54r \quad |:3 \end{cases} \Leftrightarrow \begin{cases} a_1 = 15 - 5r \\ 45 = 18r \quad |:3 \end{cases} \Leftrightarrow \begin{cases} a_1 = 15 - 5r \\ 15 = 6r \quad |:3 \end{cases} \Leftrightarrow \begin{cases} a_1 = 15 - 5r \\ 5 = 2r \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a_1 = \overset{2)}{15} - 5\cdot\frac{5}{2} \\ r = \frac{5}{2} \end{cases} \Leftrightarrow \begin{cases} a_1 = \frac{30 - 25}{2} \\ r = 5/2 \end{cases} \Leftrightarrow \begin{cases} a_1 = \frac{5}{2} \\ r = \frac{5}{2} \end{cases}$$

Aplicatie

$$\begin{cases} a_2 + a_6 + a_9 = 45 \\ a_3 + a_7 + a_{10} = 54 \end{cases} \qquad \begin{aligned} a_1 &= ? \\ r &= ? \end{aligned}$$

$$a_2 = a_1 + r, \quad a_3 = a_1 + 2r$$
$$a_6 = a_1 + 5r, \quad a_7 = a_1 + 6r$$
$$a_9 = a_1 + 8r, \quad a_{10} = a_1 + 9r$$

$$\begin{cases} a_1 + r + a_1 + 5r + a_1 + 8r = 45 \\ a_1 + 2r + a_1 + 6r + a_1 + 9r = 54 \end{cases} \Leftrightarrow \begin{cases} 3a_1 + 14r = 45 \\ 3a_1 + 17r = 54 \end{cases} \Leftrightarrow \begin{cases} 3a_1 = 45 - 14r \\ 45 - 14r + 17r = 54 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 3a_1 = 45 - 14r \\ 3r = 54 - 45 \end{cases} \Leftrightarrow \begin{cases} 3a_1 = 45 - 14r \\ 3r = 9 \end{cases} \Leftrightarrow \begin{cases} 3a_1 = 45 - 42 \\ r = 3 \end{cases} \Leftrightarrow \begin{cases} 3a_1 = 3 \\ r = 3 \end{cases} \Leftrightarrow \begin{cases} a_1 = 1 \\ r = 3 \end{cases}$$

Aplicatie

Sa se determine primul termen si ratia unei progresii aritmetice daca $\begin{cases} S_3 = 12 \\ S_6 = 51 \end{cases}$

$$\begin{cases} \frac{a_1+a_3}{2}\cdot 3 = 12 \\ \frac{a_1+a_6}{2}\cdot 6 = 51 \end{cases} \Leftrightarrow \begin{cases} (a_1+a_3)\cdot 3 = 24 \mid :3 \\ (a_1+a_6)\cdot 6 = 102 \mid :6 \end{cases} \Leftrightarrow \begin{cases} a_1+a_3 = 8 \\ a_1+a_6 = 17 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a_1+a_1+2r = 8 \\ a_1+a_1+5r = 17 \end{cases} \Leftrightarrow \begin{cases} 2a_1+2r = 8 \\ 2a_1+5r = 17 \end{cases} \Leftrightarrow \begin{cases} 2a_1 = 8-2r \mid :2 \\ 8-2r+5r = 17 \end{cases} \Leftrightarrow \begin{cases} a_1 = 4-r \\ 3r = 9 \end{cases} \Leftrightarrow \begin{cases} a_1 = 1 \\ r = 3 \end{cases}$$

$$\boxed{S_n = a_1+a_2+\dots+a_n = \frac{a_1+a_n}{2}\cdot n} \qquad \boxed{a_n = a_1+(n-1)\cdot r}$$

Aplicatie

Sa se determine x numar real astfel incat tripletul de numere x-4, x+2, 2x+2 sa fie in progresie aritmetica.

$$x+2 = \frac{(x-4)+(2x+2)}{2} \Leftrightarrow x+4 = \frac{3x-2}{2} \Leftrightarrow 2x+8 = 3x-2 \Leftrightarrow$$

$$\Leftrightarrow 8+2 = 3x-2x \Leftrightarrow \boxed{10 = x}$$

$$\boxed{a_n = \frac{a_{n-1}+a_{n+1}}{2}}$$

Aplicatie

$$\div\begin{cases} S_2 - S_4 + a_2 = 14 \\ S_3 + a_3 = 17 \end{cases} \qquad \begin{aligned} a_1 &= ? \\ r &= ? \end{aligned}$$

$$\boxed{\begin{aligned} a_n &= a_1 + (n-1)\cdot r \\ S_n &= \frac{a_1 + a_n}{2}\cdot n \end{aligned}}$$

$$\begin{cases} a_1 + a_2 - \left(\frac{a_1 + a_4}{\cancel{2}}\cdot \cancel{4}^{\,2}\right) + a_2 = 14 \\ \frac{a_1 + a_3}{2}\cdot 3 + a_3 = 17 \;\Big|\cdot 2 \end{cases} \Leftrightarrow \begin{cases} a_1 + a_2 - 2(a_1 + a_4) + a_2 = 14 \\ 3(a_1 + a_3) + 2a_3 = 34 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \cancel{a_1} + \cancel{a_1} + r - \cancel{2}a_1 - 2(a_1 + 3r) + a_1 + r = 14 \\ 3a_1 + 3(a_1 + 2r) + 2(a_1 + 2r) = 34 \end{cases} \Leftrightarrow \begin{cases} -a_1 - 4r = 14 \\ 8a_1 + 10r = 34 \end{cases} \Leftrightarrow \begin{cases} a_1 = -14 - 4r \\ 8(-14-4r) + 10r = 34 \end{cases}$$

$$\Leftrightarrow \begin{cases} a_1 = -14 - 4r \\ -112 - 32r + 10r = 34 \end{cases} \Leftrightarrow \begin{cases} a_1 = -14 - 4r \\ -22r = 146 \end{cases} \Leftrightarrow \begin{cases} a_1 = -13 - 4r \\ r = \frac{146}{-22} \end{cases} \Leftrightarrow \begin{cases} a_1 = -14 - 4\cdot\frac{73}{-11} \\ r = \frac{73}{-11} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a_1 = -14 + \frac{292}{11} \\ r = -\frac{73}{11} \end{cases} \Leftrightarrow \begin{cases} a_1 = \frac{-154 + 292}{11} \\ r = -\frac{73}{11} \end{cases} \Leftrightarrow \begin{cases} a_1 = \frac{138}{11} \\ r = -\frac{73}{11} \end{cases}$$

Progresii geometrice

Numim progresie geometrica un sir de numere cu primul termen nenul, iar termenii urmatori se obtin din termenul precedent inmultit cu un numar fixat nenul numit ratia progresiei.

Ex. $\div\!\div$ 1, 2, 4, 8, 16, ... $\qquad b_1 = 1,\ q = 2$ (ratia)

Pentru a arata ca este vorba de o progresie geometrica se pune in fata sirului semnul $\div\!\div$

De obicei termenii unei progresii geometrice se vor scrie b_n , $(b_n)_{n\in N}$

Proprietati ale progresiei geometrice

1) $b_n = b_{n-1} \cdot q \quad , q \neq 0$

2) $b_n = \sqrt{b_{n-1} \cdot b_{n+1}}$ (orice termen dupa primul este media geometrica a termenilor alaturati; de aici vine si denumirea de progresie geometrica)

3) $b_n = b_1 \cdot q^{n-1} \quad , n > 1$ (formula termenului general)

4) $S_n = b_1 + b_2 + \dots + b_n = b_1 \cdot \frac{1-q^n}{1-q} = b_1 \cdot \frac{q^n - 1}{q - 1}$

Aplicatii

Sa se determine termenul de rang n al progresiei geometrice $\underset{b_1}{6}, \underset{b_2}{18}, \underset{b_3}{54}, \dots$

$18 = 6 \cdot 3$

$54 = 18 \cdot 3 \quad \Rightarrow r = 3$

$b_n = b_1 \cdot q^{n-1}$

$b_n = 6 \cdot 3^{n-1}$ termenul general

Aplicatie

Sa se determine primii doi termeni ai progresiei geometrice $b_8 = 256,\ q = 4$.

$\div\ b_1, b_2, \dots, \underset{\underset{256}{\parallel}}{b_8}, \dots$

$256 \mid 4$
$64 \mid 4$
$16 \mid 4$
$4 \mid 4$
1

$b_8 = b_1 \cdot q^{8-1} \Leftrightarrow 256 = b_1 \cdot 4^7 \Leftrightarrow 4^4 = b_1 \cdot 4^7 \Leftrightarrow b_1 = 4^4 : 4^7 \Leftrightarrow$

$\Leftrightarrow b_1 = 4^{4-7} \Leftrightarrow b_1 = 4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

$b_2 = b_1 \cdot q = \frac{1}{64} \cdot 4 = \frac{1}{16}$

$$\boxed{b_n = b_1 \cdot q^{n-1}}$$

Aplicatie

Sa se determine suma primilor n termeni ai progresiei geometrice $b_1 = 3,\ q = 2,\ n = 6$.

$S_6 = b_1 \cdot \frac{q^6 - 1}{q - 1} = 3 \cdot \frac{2^6 - 1}{2 - 1} = 3 \cdot \frac{64 - 1}{1} =$

$= 3 \cdot 63 = 189$

$$\boxed{S_n = b_1 \cdot \frac{q^n - 1}{q - 1}}$$

Aplicatie

Sa se determine suma primilor n termeni ai progresiei geometrice $-9, -3, 1, \dots$, $n=7$.

$b_1 = -9$

$b_2 = b_1 \cdot q \Leftrightarrow -3 = -9 \cdot q \Leftrightarrow q = \frac{-3}{-9} = \frac{1}{3}$

$$S_7 = b_1 \cdot \frac{q^7-1}{q-1} = -9 \cdot \frac{\left(\frac{1}{3}\right)^7 - 1}{\frac{1}{3}-1} = -9 \cdot \frac{\frac{1}{3^7}-1}{\frac{1}{3}-1} = -9 \cdot \frac{\frac{1-3^7}{3^7}}{\frac{1-3}{3}} = -9 \cdot \frac{1-3^7}{3^7} \cdot \frac{3}{1-3} =$$

$$= -9 \cdot \frac{1-2187}{2187} \cdot \frac{3}{-2} = -9 \cdot \frac{\overset{1098}{\cancel{-2186}}}{\underset{729}{\cancel{2187}}} \cdot \frac{\cancel{3}}{\cancel{-2}} = -\cancel{9} \cdot \frac{1098}{\underset{81}{\cancel{729}}} = -\frac{\overset{122}{\cancel{1098}}}{\underset{9}{\cancel{81}}} =$$

$3^7 = 3^3 \cdot 3^3 \cdot 3^1 = 27 \cdot 27 \cdot 3 = 27 \cdot 81 = 2187$

$$= -\frac{\overset{61}{\cancel{122}}}{\underset{3}{\cancel{9}}} = -\frac{61}{3}$$

Aplicatie

Sa se decida daca este progresie geometrica un sir cu termenul general $b_n = \frac{2^n}{3^n}$.

Rezolvare: Un sir este progresie geometrica daca orice termen dupa primul este medie geometrica a termenilor alaturati

In cazul nostru de verificat $b_n = \sqrt{b_{n-1} \cdot b_{n+1}}$

$$\sqrt{b_{n-1} \cdot b_{n+1}} = \sqrt{\frac{2^{n-1}}{3^{n-1}} \cdot \frac{2^{n+1}}{3^{n+1}}} = \sqrt{\frac{2^{n-\cancel{1}+n+\cancel{1}}}{3^{n-\cancel{1}+n+\cancel{1}}}} = \sqrt{\frac{2^{2n}}{3^{2n}}} = \sqrt{\left(\frac{2^n}{3^n}\right)^2} =$$

$$= \frac{2^n}{3^n} = b_n \quad \text{c.c.t.d.}$$

Aplicatie

Sa se determine o progresie geometrica stiind ca suma primilor trei termeni este 21, iar suma urmatorilor trei termeni este 168.

Rezolvare: $\div\ \underbrace{b_1, b_2, b_3}_{21}, \underbrace{b_4, b_5, b_6}_{168}, \dots$

$$\begin{cases} b_1+b_2+b_3=21 \\ b_4+b_5+b_6=168 \end{cases} \Leftrightarrow \begin{cases} b_1+b_1\cdot q+b_1\cdot q^2=21 \\ b_1\cdot q^3+b_1\cdot q^4+b_1\cdot q^5=168 \end{cases} \Leftrightarrow \begin{cases} b_1(1+q+q^2)=21 \\ b_1\cdot q^3(1+q+q^2)=168 \end{cases} \Rightarrow$$

$$\Rightarrow \cancel{21}\cdot q^3=\cancel{168}\ |:7|:3 \Rightarrow q^3=8 \Rightarrow \underline{q=2}$$

$$b_1\cdot(1+2+2^2)=21 \Leftrightarrow b_1\cdot 7=21 \Leftrightarrow \underline{b_1=3}$$

$$b_n=b_1\cdot q^{n-1}$$