2) $1\cdot 2+2\cdot 3+\dots+n(n+1) \overset{?}{=} \frac{n(n+1)(n+2)}{3}$

Pas 1. Verificare pt. $n=1$

$1\cdot 2 \overset{?}{=} \frac{1(1+1)(1+2)}{3} \Leftrightarrow 2 \overset{?}{=} \frac{1\cdot 2\cdot \cancel{3}}{\cancel{3}} \Leftrightarrow 2=2$ (A)

Pas 2. Considerăm adevărat pt. n

Pas 3. Demonstrăm pt. $n+1$, adică

$$\underbrace{1\cdot 2+2\cdot 3+\dots+n(n+1)}_{\frac{n(n+1)(n+2)}{3}}+(n+1)(n+1+1) \overset{?}{=} \frac{(n+1)(n+2)(n+3)}{3}$$

$$\frac{n(n+1)(n+2)}{3}+(n+1)(n+2) \overset{?}{=} \frac{(n+1)(n+2)(n+3)}{3}$$

$$\frac{n(n+1)(n+2)}{3}+(n+1)(n+2)=(n+1)(n+2)\left(\frac{n}{3}+{}^{3)}1\right)=\frac{(n+1)(n+2)(n+3)}{3}$$

Prin urmare enuntul este adevarat pentru orice n numar natural diferit de 0. c.c.t.d.

3) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\dots+\frac{1}{n(n+1)} \overset{?}{=} \frac{n}{n+1}$

Pas 1. Verificare pt. $n=1$

$\frac{1}{1\cdot 2} \overset{?}{=} \frac{1}{1+1} \Leftrightarrow \frac{1}{2}=\frac{1}{2}$ (A)

Pas 2. Considerăm adevărat pt. n (ce scrie în enunț)

Pas 3. Demonstrăm pt. $n+1$:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\dots+\frac{1}{n(n+1)}+\frac{1}{(n+1)(n+2)} \overset{?}{=} \frac{n+1}{n+1+1}=\frac{n+1}{n+2}$$

$$\underbrace{\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\dots+\frac{1}{n(n+1)}}_{\frac{n}{n+1}}+\frac{1}{(n+1)(n+2)} \stackrel{?}{=} \frac{n+1}{n+2}$$

$$\frac{n}{n+1}+\frac{1}{(n+1)(n+2)} = \frac{1}{n+1}\cdot\left(\overset{n+2)}{n}+\frac{1}{n+2}\right) = \frac{1}{n+1}\cdot\frac{n(n+2)+1}{n+2} =$$

$$= \frac{1}{n+1}\cdot\frac{n^2+2n+1}{n+2} = \frac{1}{\cancel{n+1}}\cdot\frac{(n+1)^{\cancel{2}}}{n+2} = \frac{n+1}{n+2} \quad \text{c.c.t.d.}$$

4) $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{(2n-1)(2n+1)} \stackrel{?}{=} \frac{n}{2n+1}$

Pas 1. Verificare pt. $n=1$

$$\frac{1}{1\cdot 3} \stackrel{?}{=} \frac{1}{2\cdot 1+1} \iff \frac{1}{3}=\frac{1}{3} \ (A)$$

Pas 2. Considerăm adevărat enunțul pt. n

Pas 3. Demonstrăm pt. $n+1$:

$$\underbrace{\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\dots+\frac{1}{(2n-1)(2n+1)}}_{\frac{n}{2n+1}}+\frac{1}{(2(n+1)-1)(2(n+1)+1)} \stackrel{?}{=} \frac{n+1}{2(n+1)+1}$$

$$\frac{n}{2n+1}+\frac{1}{(2(n+1)-1)(2(n+1)+1)} = \frac{n}{2n+1}+\frac{1}{(2n+2-1)(2n+2+1)} =$$

$$= \frac{n}{2n+1}+\frac{1}{(2n+1)(2n+3)} = \frac{1}{(2n+1)}\left(\overset{2n+3)}{n}+\frac{1}{2n+3}\right) = \frac{2n^2+3n+1}{(2n+1)(2n+3)}$$

$$\stackrel{?}{=} \frac{n+1}{2n+3}$$

$$(2n+1)(n+1) = 2n^2+2n+n+1 = 2n^2+3n+1$$

$$\frac{2n^2+3n+1}{(2n+1)(2n+3)} = \frac{\cancel{(2n+1)}(n+1)}{\cancel{(2n+1)}(2n+3)} = \frac{n+1}{2n+3} \quad \text{c.c.t.d.}$$

6) $\frac{1}{1\cdot4}+\frac{1}{4\cdot7}+\dots+\frac{1}{(3n-2)\cdot(3n+1)} = \frac{n}{3n+1}$, $\forall n \in \mathbb{N}^*$

Pas 1. Verificăm enunțul pentru $n = 1$

$$\frac{1}{1\cdot4} \overset{?}{=} \frac{1}{3\cdot1+1} \quad (A)$$

Pas 2. Considerăm adevărat enunțul pt. n

Pas 3. Demonstrăm pentru $n+1$:

$$\frac{1}{1\cdot4}+\frac{1}{4\cdot7}+\dots+\frac{1}{(3n-2)(3n+1)}+\frac{1}{(3(n+1)-2)(3\cdot(n+1)+1)} \overset{?}{=} \frac{n+1}{3(n+1)+1}$$

$$\frac{1}{1\cdot4}+\frac{1}{4\cdot7}+\dots+\frac{1}{(3n-2)(3n+1)}+\frac{1}{(3n+1)(3n+4)} \overset{?}{=} \frac{n+1}{3n+4}$$

$$\underbrace{\frac{1}{1\cdot4}+\frac{1}{4\cdot7}+\dots+\frac{1}{(3n-2)(3n+1)}}_{\frac{n}{3n+1}}+\frac{1}{(3n+1)(3n+4)} \overset{?}{=} \boxed{\frac{n+1}{3n+4}}$$

$$\frac{n}{3n+1}+\frac{1}{(3n+1)(3n+4)} = \frac{1}{3n+1}\cdot\left(n+\frac{1}{3n+4}\right) = \frac{1}{3n+1}\cdot\frac{3n^2+4n+1}{3n+4}$$

$$= \frac{3n^2+n+3n+1}{(3n+1)(3n+4)} = \frac{n(3n+1)+3n+1}{(3n+1)(3n+4)} = \frac{\cancel{(3n+1)}(n+1)}{\cancel{(3n+1)}(3n+4)} = \boxed{\frac{n+1}{3n+4}}$$

c.c.t.d.

7) $\left(1-\frac{1}{2^2}\right)\cdot\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{(n+1)^2}\right) \overset{?}{=} \frac{n+2}{2n+2}$, $\forall n \in \mathbb{N}^*$

Pas 1. Verificam pt. $n = 1$

$$1-\frac{1}{2^2} \overset{?}{=} \frac{1+2}{2\cdot1+2} \iff \overset{4)}{1}-\frac{1}{4} = \frac{3}{4} \iff \frac{3}{4} = \frac{3}{4} \quad (A)$$

Verificam și pt $n = 2$

$$\left(1-\frac{1}{2^2}\right)\cdot\left(1-\frac{1}{3^2}\right) \overset{?}{=} \frac{2+2}{2\cdot2+2} \iff \frac{\cancel{3}}{\cancel{4}}\cdot\frac{\cancel{8}^{\,2}}{\cancel{9}_{\,3}} \overset{?}{=} \frac{\cancel{4}^{\,2}}{\cancel{6}_{\,3}} \iff \frac{2}{3} = \frac{2}{3} \quad (A)$$

Pas 2. Considerăm adevărat enunțul pt. n

Pas 3. Demonstrăm pt. $(n+1)$:

$$\underbrace{\left(1-\frac{1}{2^2}\right)\cdot\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{(n+1)^2}\right)}_{\frac{n+2}{2n+2}}\cdot\left(1-\frac{1}{(n+2)^2}\right) \overset{?}{=} \frac{(n+1)+2}{2(n+1)+2} = \boxed{\frac{n+3}{2n+4}}$$

$$\frac{n+2}{2n+2}\cdot\left(1-\frac{1}{(n+2)^2}\right) = \frac{\cancel{n+2}}{2n+2}\cdot\frac{(n+2)^2-1}{(n+2)^{\cancel{2}}} = \frac{(n+2-1)(n+2+1)}{(2n+2)(n+2)} =$$

$$= \frac{\cancel{(n+1)}(n+3)}{2\cancel{(n+1)}(n+2)} = \frac{n+3}{2(n+2)} = \boxed{\frac{n+3}{2n+4}}$$ c.c.t.d.

Aplicatie

$$6+24+60+\ldots+n(n^2-1) \overset{?}{=} \boxed{\frac{n(n+1)[n^2+n-2]}{4}},\ \forall n\in\mathbb{N},\ n\geq 2.$$

Pas 1. Verificăm enunțul în primul caz, $n=2$

$$6 \overset{?}{=} \frac{2(2+1)(2^2+2-2)}{4} \Leftrightarrow 6 \overset{?}{=} \frac{2\cdot 3\cdot \cancel{4}}{\cancel{4}} \Leftrightarrow 6 \overset{?}{=} 6 \ (A)$$

Pas 2. Considerăm adevarat enunțul pentru n

Pas 3. Demonstrăm enunțul pentru $(n+1)$

$$\underbrace{6+24+\ldots+n(n^2-1)}_{\frac{n(n+1)(n^2+n-2)}{4}}+(n+1)((n+1)^2-1) \overset{?}{=} \frac{(n+1)(n+2)((n+1)^2+(n+1)-2)}{4}$$

$$\frac{n(n+1)(n^2+n-2)}{4}+(n+1)((n+1)^2-1) \overset{?}{=} \frac{(n+1)(n+2)((n+1)^2+(n+1)-2)}{4}$$

$$\frac{n(n+1)(n^2+n-2)}{4}+(n+1)(n+\cancel{1}-\cancel{1})(n+1+1) = \frac{\boxed{n(n+1)}(n^2+n-2)}{4}+$$

$$+\boxed{(n+1)\cdot n}(n+2) = n(n+1)\left[\frac{n^2+n-2}{4}+{}^{4)}(n+2)\right] =$$

$$= \frac{n}{4}(n+1)(n^2+n-2+4n+8) = \boxed{\frac{n}{4}(n+1)(n^2+5n+6)}$$

$$\frac{(n+1)}{4}(n+2)(n^2+2n+\cancel{1}+n+\cancel{1}-\cancel{2}) = \frac{n+1}{4}\cdot(n+2)\cdot n(n+3)$$

$$= \boxed{\frac{n}{4}(n+1)(n^2+5n+6)}$$ c.c.t.d.

$a^2-b^2=(a-b)(a+b)$ $\Big\}$ $(n+2)(n+3) = n^2+3n+2n+6 =$
$= n^2+5n+6$