Sapt. 12 (2-4 dec)

Aplicatie

$\begin{cases} a_4 + a_8 = 30 \\ 10 \cdot a_1 - 4 \cdot a_7 = -45 \end{cases}$ / $a_1 = ?$, $r = ?$ /

$a_4 = a_1 + (4-1)r = a_1 + 3r$

$a_8 = a_1 + (8-1)r = a_1 + 7r$

$a_7 = a_1 + (7-1)r = a_1 + 6r$

$\begin{cases} a_1 + 3r + a_1 + 7r = 30 \\ 10 \cdot a_1 - 4(a_1 + 6r) = -45 \end{cases} \Leftrightarrow \begin{cases} 2a_1 + 10r = 30 \\ 10 \cdot a_1 - 4 \cdot a_1 - 24r = -45 \end{cases} \Leftrightarrow \begin{cases} 2a_1 + 10r = 30 \mid :2 \\ 6a_1 - 24r = -45 \end{cases} \Leftrightarrow$

$\begin{cases} a_1 + 5r = 15 \\ 6a_1 - 24r = -45 \end{cases} \Leftrightarrow \begin{cases} a_1 = 15 - 5r \\ 6(15 - 5r) - 24r = -45 \end{cases} \Leftrightarrow \begin{cases} a_1 = 15 - 5r \\ 90 - 30r - 24r = -45 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} a_1 = 15 - 5r \\ 135 = 54r \mid :3 \end{cases} \Leftrightarrow \begin{cases} a_1 = 15 - 5r \\ 45 = 18r \mid :3 \end{cases} \Leftrightarrow \begin{cases} a_1 = 15 - 5r \\ 15 = 6r \mid :3 \end{cases} \Leftrightarrow \begin{cases} a_1 = 15 - 5r \\ 5 = 2r \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} a_1 = {}^{2)}15 - 5 \cdot \frac{5}{2} \\ r = \frac{5}{2} \end{cases} \Leftrightarrow \begin{cases} a_1 = \frac{30 - 25}{2} \\ r = 5/2 \end{cases} \Leftrightarrow \begin{cases} a_1 = \frac{5}{2} \\ r = \frac{5}{2} \end{cases}$

Aplicatie

$\begin{cases} a_2 + a_6 + a_9 = 45 \\ a_3 + a_7 + a_{10} = 54 \end{cases}$ / $a_1 = ?$, $r = ?$ /

$a_2 = a_1 + r$, $a_3 = a_1 + 2r$

$a_6 = a_1 + 5r$, $a_7 = a_1 + 6r$

$a_9 = a_1 + 8r$, $a_{10} = a_1 + 9r$

$\begin{cases} a_1 + r + a_1 + 5r + a_1 + 8r = 45 \\ a_1 + 2r + a_1 + 6r + a_1 + 9r = 54 \end{cases} \Leftrightarrow \begin{cases} 3a_1 + 14r = 45 \\ 3a_1 + 17r = 54 \end{cases} \Leftrightarrow \begin{cases} 3a_1 = 45 - 14r \\ 45 - 14r + 17r = 54 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 3a_1 = 45 - 14r \\ 3r = 54 - 45 \end{cases} \Leftrightarrow \begin{cases} 3a_1 = 45 - 14r \\ 3r = 9 \end{cases} \Leftrightarrow \begin{cases} 3a_1 = 45 - 42 \\ r = 3 \end{cases} \Leftrightarrow \begin{cases} 3a_1 = 3 \\ r = 3 \end{cases} \Leftrightarrow \begin{cases} a_1 = 1 \\ r = 3 \end{cases}$

Aplicatie

Sa se determine primul termen si ratia unei progresii aritmetice daca $\begin{cases} S_3 = 12 \\ S_6 = 51 \end{cases}$

$$\begin{cases} \frac{a_1 + a_3}{2} \cdot 3 = 12 \\ \frac{a_1 + a_6}{2} \cdot 6 = 51 \end{cases} \Leftrightarrow \begin{cases} (a_1 + a_3) \cdot 3 = 24 \mid :3 \\ (a_1 + a_6) \cdot 6 = 102 \mid :6 \end{cases} \Leftrightarrow \begin{cases} a_1 + a_3 = 8 \\ a_1 + a_6 = 17 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a_1 + a_1 + 2r = 8 \\ a_1 + a_1 + 5r = 17 \end{cases} \Leftrightarrow \begin{cases} 2a_1 + 2r = 8 \\ 2a_1 + 5r = 17 \end{cases} \Leftrightarrow \begin{cases} 2a_1 = 8 - 2r \mid :2 \\ 8 - 2r + 5r = 17 \end{cases} \Leftrightarrow \begin{cases} a_1 = 4 - r \\ 3r = 9 \end{cases} \Leftrightarrow \begin{cases} a_1 = 1 \\ r = 3 \end{cases}$$

$$\boxed{S_n = a_1 + a_2 + \dots + a_n = \frac{a_1 + a_n}{2} \cdot n} \qquad \boxed{a_n = a_1 + (n-1) \cdot r}$$

Aplicatie

Sa se determine x numar real astfel incat tripletul de numere x-4, x+2, 2x+2 sa fie in progresie aritmetica.

$$x + 2 = \frac{(x-4) + (2x+2)}{2} \Leftrightarrow x + 4 = \frac{3x - 2}{2} \Leftrightarrow 2x + 8 = 3x - 2 \Leftrightarrow$$

$$\Leftrightarrow 8 + 2 = 3x - 2x \Leftrightarrow \boxed{10 = x}$$

$$\boxed{a_n = \frac{a_{n-1} + a_{n+1}}{2}}$$

Aplicatie

$$\begin{cases} a_n = a_1 + (n-1)\cdot r \\ S_n = \dfrac{a_1+a_n}{2}\cdot n \end{cases}$$

$$\div\begin{cases} S_2 - S_4 + a_2 = 14 \\ S_3 + a_3 = 17 \end{cases} \qquad a_1 = ? \quad r = ?$$

$$\begin{cases} a_1+a_2-\left(\dfrac{a_1+a_4}{\cancel{2}}\cdot \cancel{4}^{\,2}\right)+a_2 = 14 \\ \dfrac{a_1+a_3}{2}\cdot 3 + a_3 = 17 \;\Big|\cdot 2 \end{cases} \Leftrightarrow \begin{cases} a_1 + a_2 - 2(a_1+a_4) + a_2 = 14 \\ 3(a_1+a_3) + 2a_3 = 34 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \cancel{a_1} + \cancel{a_1} + r - \cancel{2}a_1 - 2(a_1+3r) + a_1 + r = 14 \\ 3a_1 + 3(a_1+2r) + 2(a_1+2r) = 34 \end{cases} \Leftrightarrow \begin{cases} -a_1 - 4r = 14 \\ 8a_1 + 10r = 34 \end{cases} \Leftrightarrow \begin{cases} a_1 = -14-4r \\ 8(-14-4r)+10r = 34 \end{cases}$$

$$\Leftrightarrow \begin{cases} a_1 = -14-4r \\ -112-32r+10r = 34 \end{cases} \Leftrightarrow \begin{cases} a_1 = -14-4r \\ -22r = 146 \end{cases} \Leftrightarrow \begin{cases} a_1 = -13-4r \\ r = \dfrac{146}{-22} \end{cases} \Leftrightarrow \begin{cases} a_1 = -14-4\cdot\dfrac{73}{-11} \\ r = \dfrac{73}{-11} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a_1 = -14 + \dfrac{292}{11} \\ r = -\dfrac{73}{11} \end{cases} \Leftrightarrow \begin{cases} a_1 = \dfrac{-154+292}{11} \\ r = -\dfrac{73}{11} \end{cases} \Leftrightarrow \begin{cases} a_1 = \dfrac{138}{11} \\ r = -\dfrac{73}{11} \end{cases}$$

Aplicatii

Sa se determine termenul de rang n al progresiei geometrice 6, 18, 54, ...

b_1, b_2, b_3

$$18 = 6\cdot 3$$
$$54 = 18\cdot 3 \quad \Rightarrow r = 3$$

$$\boxed{b_n = b_1 \cdot q^{n-1}}$$

$b_n = 6\cdot 3^{n-1}$ termenul general

Aplicatie

Sa se determine primii doi termeni ai progresiei geometrice $b_8 = 256, \quad q = 4$.

$\div\, b_1, b_2, \dots, \underset{\underset{256}{\parallel}}{b_8}, \dots$

$$\begin{array}{r|l} 256 & 4 \\ 64 & 4 \\ 16 & 4 \\ 4 & 4 \\ 1 & \end{array}$$

$b_8 = b_1 \cdot q^{8-1} \Leftrightarrow 256 = b_1 \cdot 4^7 \Leftrightarrow 4^4 = b_1 \cdot 4^7 \Leftrightarrow b_1 = 4^4 : 4^7 \Leftrightarrow$

$\Leftrightarrow b_1 = 4^{4-7} \Leftrightarrow b_1 = 4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

$b_2 = b_1 \cdot q = \frac{1}{64} \cdot 4 = \frac{1}{16}$

$$\boxed{b_n = b_1 \cdot q^{n-1}}$$

Aplicatie

Sa se determine suma primilor n termeni ai progresiei geometrice $b_1 = 3, \quad q = 2, \quad n = 6$.

$S_6 = b_1 \cdot \frac{q^6 - 1}{q - 1} = 3 \cdot \frac{2^6 - 1}{2 - 1} = 3 \cdot \frac{64 - 1}{1} =$

$= 3 \cdot 63 = 189$

$$\boxed{S_n = b_1 \cdot \frac{q^n - 1}{q - 1}}$$

Aplicatie

Sa se determine suma primilor n termeni ai progresiei geometrice $-9, -3, 1, \dots$, $n = 7$.

$$\underset{b_1}{-9},\ \underset{b_2}{-3},\ \underset{b_3}{1},\ \dots$$

$b_1 = -9$

$b_2 = b_1 \cdot q \Leftrightarrow -3 = -9 \cdot q \Leftrightarrow q = \frac{-3}{-9} = \frac{1}{3}$

$$S_7 = b_1 \cdot \frac{q^7 - 1}{q - 1} = -9 \cdot \frac{\left(\frac{1}{3}\right)^7 - 1}{\frac{1}{3} - 1} = -9 \cdot \frac{\frac{1}{3^7} - 1}{\frac{1}{3} - 1} = -9 \cdot \frac{\frac{1 - 3^7}{3^7}}{\frac{1-3}{3}} = -9 \cdot \frac{1 - 3^7}{3^7} \cdot \frac{3}{1-3} =$$

$$= -9 \cdot \frac{1 - 2187}{2187} \cdot \frac{3}{-2} = -9 \cdot \frac{\overset{1098}{\cancel{-2186}}}{\underset{729}{\cancel{2187}}} \cdot \frac{\cancel{3}}{\cancel{-2}} = -\cancel{9} \cdot \frac{1098}{\underset{81}{\cancel{729}}} = -\frac{\overset{129}{\cancel{1098}}}{\underset{9}{\cancel{81}}} =$$

$3^7 = 3^3 \cdot 3^3 \cdot 3^1 = 27 \cdot 27 \cdot 3 = 27 \cdot 81 = 2187$

$$= -\frac{\overset{43}{\cancel{129}}}{\underset{3}{\cancel{9}}} = -\frac{43}{3}$$

Aplicatie

Sa se decida daca este progresie geometrica un sir cu termenul general $b_n = \frac{2^n}{3^n}$.

Rezolvare: Un sir este progresie geometrica daca orice termen dupa primul este medie geometrica a termenilor alaturati

In cazul nostru de verificat $b_n = \sqrt{b_{n-1} \cdot b_{n+1}}$

$$\sqrt{b_{n-1} \cdot b_{n+1}} = \sqrt{\frac{2^{n-1}}{3^{n-1}} \cdot \frac{2^{n+1}}{3^{n+1}}} = \sqrt{\frac{2^{n-\cancel{1}+n+\cancel{1}}}{3^{n-\cancel{1}+n+\cancel{1}}}} = \sqrt{\frac{2^{2n}}{3^{2n}}} = \sqrt{\left(\frac{2^n}{3^n}\right)^2} =$$

$$= \frac{2^n}{3^n} = b_n \quad \text{c.c.t.d.}$$

Aplicatie

Sa se determine o progresie geometrica stiind ca suma primilor trei termeni este 21, iar suma urmatorilor trei termeni este 168.

Rezolvare: $\div\ \underbrace{b_1, b_2, b_3}_{21}, \underbrace{b_4, b_5, b_6}_{168}, \ldots$

$$\begin{cases} b_1+b_2+b_3=21 \\ b_4+b_5+b_6=168 \end{cases} \Leftrightarrow \begin{cases} b_1+b_1\cdot q+b_1\cdot q^2=21 \\ b_1\cdot q^3+b_1\cdot q^4+b_1\cdot q^5=168 \end{cases} \Leftrightarrow \begin{cases} b_1(1+q+q^2)=21 \\ b_1\cdot q^3(1+q+q^2)=168 \end{cases} \Rightarrow$$

$$\Rightarrow \cancel{21}^{\,\cancel{7}\,1}\cdot q^3=\cancel{168}^{\,\cancel{24}\,8} \ \Big|:7\Big|:3 \Rightarrow q^3=8 \Rightarrow \boxed{q=2}$$

$$b_1\cdot(1+2+2^2)=21 \Leftrightarrow b_1\cdot 7=21 \Leftrightarrow \boxed{b_1=3}$$

$$b_n=b_1\cdot q^{n-1}$$

Aplicatie

Sa se determine ratia si suma primilor n termeni ai progresiei aritmetice $a_1=-10,\ a_n=-20,\ n=6.$

Rezolvare

$$S_6=\frac{a_1+a_6}{2}\cdot 6=\frac{-10-20}{2}\cdot 6=\frac{-30}{\cancel{2}_1}\cdot\cancel{6}^3=\boxed{-90}$$

$$n=6 \Rightarrow a_n=a_6=a_1+(6-1)\cdot r \Rightarrow -20=-10+5\cdot r \Rightarrow -10=5\cdot r \Rightarrow \boxed{r=-2}$$

$$S_n=\frac{a_1+a_n}{2}\cdot n$$

$$a_n=a_1+(n-1)\cdot r$$

Aplicatie

$b_2 = b_1 \cdot q \quad , \quad b_3 = b_1 \cdot q^2 \quad , \quad b_4 = b_1 \cdot q^3$

$\begin{cases} b_2(b_3 - b_1) = 24 \\ b_3(b_4 - b_2) = 96 \end{cases} \quad \div \quad \begin{matrix} b_1 = ? \\ q = ? \end{matrix}$

$\begin{cases} b_1 \cdot q\,(b_1 \cdot q^2 - b_1) = 24 \\ b_1 \cdot q^2\,(b_1 \cdot q^3 - b_1 \cdot q) = 96 \end{cases} \Leftrightarrow \begin{cases} b_1^2 \cdot q \cdot (q^2 - 1) = 24 \\ b_1^2 \cdot q^3\,(q^2 - 1) = 96 \end{cases} \Leftrightarrow \begin{cases} b_1^2\, q\,(q^2-1) = 24 \\ \underbrace{b_1^2\, q\,(q^2-1)}_{24} \cdot q^2 = 96 \end{cases}$

$24 \cdot q^2 = 96 \;\Big|\, :24$

$q^2 = 4$

$\boxed{q = \pm 2}$

$\begin{cases} q = 2 \\ b_1^2 \cdot 2\,(2^2 - 1) = 24 \end{cases} \Leftrightarrow \begin{cases} q = 2 \\ b_1^2 = 4 \end{cases} \Rightarrow b_1 = \pm 2$

$\begin{cases} q = -2 \\ b_1^2 \cdot (-2)\,((-2)^2 - 1) = 24 \end{cases} \Leftrightarrow \begin{cases} q = -2 \\ b_1^2 = -4 \text{, imposibil} \end{cases}$

$b_n = b_1 \cdot q^{n-1}$

Soluția: $\begin{cases} q = 2 \\ b_1 = \pm 2 \end{cases}$

Aplicatie

Fie punctele A(2,3), B(-3,3), C(1,-2). Sa se determine coordonatele vectorilor $\overrightarrow{AB}$, $\overrightarrow{BC}$.

Rezolvare:

$\overrightarrow{AB} = (x_B - x_A\,,\, y_B - y_A) = (-3-2\,,\, 3-3) = (-5\,,\,0) = -5\,\vec{i}$

$\overrightarrow{BC} = (x_C - x_B\,,\, y_C - y_B) = (1-(-3)\,,\, -2-3) = (4\,,\,-5) = 4 \cdot \vec{i} - 5 \cdot \vec{j}$

$\overrightarrow{AB} = (x_B - x_A) \cdot \vec{i} + (y_B - y_A) \cdot \vec{j} = (x_B - x_A\,,\, y_B - y_A)$

Aplicatie

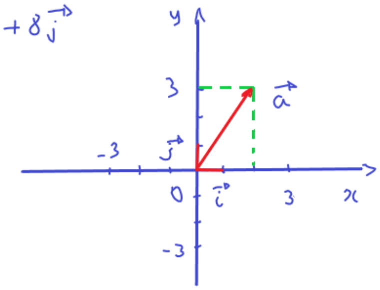
Fie vectorii $\vec{a} = 2\vec{i} + 3\vec{j}$, $\vec{b} = -\vec{i} - 2\vec{j}$, $\vec{c} = 3\vec{i} - 2\vec{j}$.

Sa se calculeze $\vec{v} = 2\vec{a} - 3\vec{b} + 2\vec{c}$.

Rezolvare:

$\vec{v} = 2\vec{a} - 3\vec{b} + 2\vec{c} = 2 \cdot (2\vec{i} + 3\vec{j}) - 3 \cdot (-\vec{i} - 2\vec{j}) + 2 \cdot (3\vec{i} - 2\vec{j}) =$

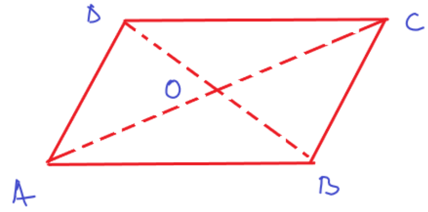
$= 4\vec{i} + 6\vec{j} + 3\vec{i} + 6\vec{j} + 6\vec{i} - 4\vec{j} = 13\vec{i} + 8\vec{j}$



Aplicatie

Fie paralelogramul ABCD si O mijlocul sau. Sa se exprime sumele $\vec{AB}+\vec{OD}$ si $\vec{AC}+\vec{CD}$.

Rezolvare



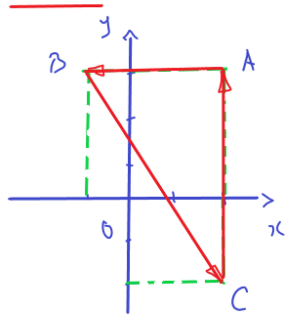
$$\vec{AB}+\vec{OD} \overset{\vec{OD}=\vec{BO}}{=} \vec{AB}+\vec{BO} \underset{\Delta}{\overset{\text{regula}}{=}} \vec{AC}$$

$$\vec{AC}+\vec{CD} \underset{\Delta}{\overset{\text{regula}}{=}} \vec{AD}$$

Aplicatie

Fie punctele A(2,3), B(-1,3), C(2,-2). Sa se reprezinte grafic vectorii $\vec{AB}$, $\vec{BC}$, $\vec{CA}$ si sa se afle modulele lor.

Rezolvare



$$|\vec{AB}| = \sqrt{(x_B-x_A)^2+(y_B-y_A)^2} = \sqrt{(-1-2)^2+(3-3)^2} = \sqrt{9+0} = 3$$

$$|\vec{BC}| = \sqrt{(x_C-x_B)^2+(y_C-y_B)^2} = \sqrt{(2-(-1))^2+(-2-3)^2} = \sqrt{9+25} = \sqrt{34}$$

$$|\vec{CA}| = \sqrt{(x_A-x_C)^2+(y_A-y_C)^2} = \sqrt{(2-2)^2+(3-(-2))^2} = \sqrt{0+25} = 5$$

Lucrare de control

Oficiu: 4p

1) Sa se demonstreze prin metoda inductiei matematice ca pentru $n \in \mathbb{N}^*$ are loc egalitatea:

1p $$\frac{1}{4\cdot 7}+\frac{1}{4\cdot 7}+\dots+\frac{1}{(3n-2)\cdot(3n+1)}=\frac{n}{3n+1}$$

2) Sa se determine ratia si suma primilor n termeni ai progresiei aritmetice (a_n) daca:

1p $$a_3=12,\quad a_{14}=144,\quad n=15$$

3) Sa se determine ratia si primul termen al progresiei geometrice (b_n) daca: $\begin{cases} b_1+b_2+b_3=14 \\ b_2+b_3+b_4=28 \end{cases}$.

1p

4) Sa se reprezinte grafic vectorii $\vec{AB}$ si $\vec{CD}$, unde A(0,-2), B(-2,-1), C(2,2). Determinati modulele vectorilor.

1p

5) Se considera vectorii $\vec{a}=2\cdot\vec{i}-3\cdot\vec{j}$, $\vec{b}=\vec{i}+2\cdot\vec{j}$, $\vec{c}=-\vec{i}+3\cdot\vec{j}$. Sa se determine coordonatele vectorului $\vec{a}+2\cdot\vec{b}-\vec{c}$.

1p

6) Se considera paralelogramul ABCD in plan si se noteaza cu O centrul sau.

1p Sa se exprime sumele $\vec{AB}+\vec{CO}$ si $\vec{AB}+\vec{CO}+\vec{OD}$.

Aplicatie

$$S_n = \frac{a_1 + a_n}{2} \cdot n$$
$$a_n = a_1 + (n-1) \cdot r$$

Sa se rezolve ecuatia $1 + 4 + 7 + \dots + x = 117$

Rezolvare:

Observam ca termenii sunt in progresie aritmetica $a_1 = 1$, $r = 3$

$$\begin{cases} S_n = \frac{1+x}{2} \cdot n = 117 \\ x = 1 + (n-1)\cdot 3 \end{cases} \Leftrightarrow \begin{cases} n(1+x) = 234 \\ x = 1 + 3n - 3 \end{cases} \Leftrightarrow \begin{cases} n(1+3n-2) = 234 \\ x = 3n - 2 \end{cases}$$

$$\Leftrightarrow \begin{cases} n = 9 \\ x = 3\cdot 9 - 2 \end{cases} \Leftrightarrow \begin{cases} n = 9 \\ x = 25 \end{cases}$$

$$n(1+3n-2) = 234$$
$$n(3n-1) = 234$$
$$3n^2 - n - 234 = 0$$

$$\Delta = (-1)^2 - 4\cdot 3\cdot(-234)$$
$$\Delta = 1 + 2808$$
$$\Delta = 2809 = 53^2$$

$$n_1 = \frac{1+53}{6} = \frac{54}{6} = \boxed{9}$$
$$n_2 = \frac{1-53}{6} = \frac{-52}{6} \notin \mathbb{N}$$

Aplicatie

Sa se determine x numar real astfel incat tripletul sa fie format din numere in progresie geometrica

$3x-1$, $x+3$, $9-x$.

Rezolvare: $\div\div\ b_1, b_2, \dots, b_{n-1}, b_n, b_{n+1}, \dots$ $\qquad b_n = \sqrt{b_{n-1} \cdot b_{n+1}}$

$$x+3 = \sqrt{(3x-1)(9-x)} \Rightarrow (x+3)^2 = (3x-1)(9-x) \Leftrightarrow$$

$$\Leftrightarrow x^2 + 6x + 9 = 27x - 3x^2 - 9 + x \Leftrightarrow x^2 + 6x + 9 - 27x + 3x^2 + 9 - x = 0$$

$$\Leftrightarrow 4x^2 - 22x + 18 = 0 \mid :2 \Rightarrow 2x^2 - 11x + 9 = 0 \Rightarrow 2x^2 - 2x - 9x + 9 = 0$$

$$\Leftrightarrow 2x(x-1) - 9(x-1) = 0 \Leftrightarrow (x-1)(2x-9) = 0 \Leftrightarrow x - 1 = 0 \text{ sau } 2x - 9 = 0$$

$$\Leftrightarrow x = 1 \text{ sau } x = \frac{9}{2}$$

Aplicatie

Sa se rezolve ecuatia $(3x-1) + (3x-4) + (3x-7) + \dots + (3x-58) = 790$

Rezolvare:

Observam ca se aduna termenii unei progresii aritmetice de ratie -3.

$$\begin{cases} S_n = 790 \\ a_n = a_1 + (n-1)\cdot r \end{cases} \Leftrightarrow \begin{cases} \frac{(3x-1)+(3x-58)}{2} \cdot n = 790 \\ \cancel{3x} - 58 = \cancel{3x} - 1 + (n-1)(-3) \end{cases} \Leftrightarrow \begin{cases} \frac{6x-59}{2}\cdot n = 790 \\ -58 = -1 - 3(n-1) \end{cases}$$

$$\Leftrightarrow \begin{cases} (6x-59)\cdot n = 1580 \\ 3(n-1) = 57 \mid :3 \end{cases} \Leftrightarrow \begin{cases} (6x-59)\cdot n = 1580 \\ n - 1 = 19 \end{cases} \Leftrightarrow \begin{cases} (6x-59)\cdot 20 = 1580 \mid :20 \\ n = 20 \end{cases}$$

$$\Leftrightarrow \begin{cases} 6x - 59 = 79 \\ n = 20 \end{cases} \Leftrightarrow \begin{cases} 6x = 138 \\ n = 20 \end{cases} \Leftrightarrow \begin{cases} x = 23 \\ n = 20 \end{cases}$$

Aplicatie

Sa se determine primul termen si ratia progresiei geometrice $\begin{cases} b_5 - b_1 = 30 \\ b_4 - b_2 = 12 \end{cases}$.

$$\begin{cases} b_1 \cdot q^{5-1} - b_1 = 30 \\ b_1 \cdot q^{4-1} - b_1 \cdot q^{2-1} = 12 \end{cases} \Leftrightarrow \begin{cases} b_1 q^4 - b_1 = 30 \\ b_1 \cdot q^3 - b_1 \cdot q = 12 \end{cases} \Leftrightarrow \begin{cases} b_1 (q^4 - 1) = 30 \\ b_1 q (q^2 - 1) = 12 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} b_1 = \frac{30}{q^4 - 1} \\ \frac{30}{q^4-1} \cdot q(q^2-1) = 12 \end{cases}$$

$$\frac{30}{\cancel{(q^2-1)}(q^2+1)} \cdot q \cdot \cancel{(q^2-1)} = 12 \Leftrightarrow \frac{30 \cdot q}{q^2+1} = 12 \,|:6 \Leftrightarrow \frac{5q}{q^2+1} = 2 \Leftrightarrow 5q = 2q^2 + 2$$

$$b_n = b_1 \cdot q^{n-1} \qquad \Leftrightarrow 2q^2 - 5q + 2 = 0$$

$$\underset{a}{\overset{\uparrow}{2}}q^2 \underset{b}{\overset{\uparrow}{-5}}q + \underset{c}{\overset{\uparrow}{2}} = 0$$

$$\Delta = b^2 - 4ac = (-5)^2 - 4 \cdot 2 \cdot 2 =$$
$$= 25 - 16 = 9$$

$$q_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{5 + \sqrt{9}}{4} = \frac{5+3}{4} = 2$$

$$q_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{5-3}{4} = \frac{2}{4} = \frac{1}{2}$$

Dacă $q = 2$ atunci $b_1 = \frac{30}{q^4 - 1} = \frac{30}{2^4 - 1} = \frac{30}{16-1} = \frac{30}{15} = 2$.

Dacă $q = \frac{1}{2}$ atunci $b_1 = \frac{30}{q^4-1} = \frac{30}{\frac{1}{16} - 1} = \frac{30}{\frac{1-16}{16}} = \frac{16 \cdot 30}{-15} = 16 \cdot (-2) = -32$

Aplicatie

Sa se determine primul termen ratia progresiei geometrice $\begin{cases} b_2 + b_5 - b_4 = 10 \\ b_3 + b_6 - b_5 = 20 \end{cases}$.

$$\begin{cases} b_1 \cdot q^{2-1} + b_1 \cdot q^{5-1} - b_1 \cdot q^{4-1} = 10 \\ b_1 \cdot q^{3-1} + b_1 \cdot q^{6-1} - b_1 \cdot q^{5-1} = 20 \end{cases} \Leftrightarrow \begin{cases} b_1 \cdot (q + q^4 - q^3) = 10 \\ b_1 \cdot (q^2 + q^5 - q^4) = 20 \end{cases} \Leftrightarrow \begin{cases} b_1 \cdot (q + q^4 - q^3) = 10 \\ q \cdot \underbrace{b_1 (q + q^4 - q^3)}_{10} = 20 \end{cases}$$

$$\begin{cases} b_1 (q + q^4 - q^3) = 10 \\ q \cdot 10 = 20 \end{cases} \Leftrightarrow \begin{cases} b_1 \cdot (2 + 2^4 - 2^3) = 10 \\ q = 2 \end{cases} \Leftrightarrow \begin{cases} b_1 \cdot (2 + 16 - 8) = 10 \\ q = 2 \end{cases} \Leftrightarrow$$

$$\begin{cases} b_1 \cdot 10 = 10 \\ q = 2 \end{cases} \Leftrightarrow \begin{cases} b_1 = 1 \\ q = 2 \end{cases}$$

Aplicatie

Sa se determine suma primilor n termeni ai progresiei geometrice $b_1 = 2,5$, $q = 1,5$, $n = 5$.

$$S_5 = b_1 + b_2 + b_3 + b_4 + b_5 = b_1 \cdot \frac{q^5 - 1}{q - 1}$$

$$S_5 = 2,5 \cdot \frac{1,5^5 - 1}{1,5 - 1} = 2,5 \cdot \frac{7,59375 - 1}{0,5} = 2,5 \cdot 13,1875 = 32,96875$$

Aplicatie

Sa se determine primii doi termeni ai progresiei geometrice $b_8 = 256$, $q = 4$

$$b_n = b_1 \cdot q^{n-1} \Rightarrow b_8 = b_1 \cdot q^{8-1} \Rightarrow 256 = b_1 \cdot 4^7 \Rightarrow \boxed{b_1 = \frac{256}{4^7} = \frac{4^4}{4^7} = \frac{1}{4^3} = \frac{1}{64}}$$

$$\boxed{b_2 = b_1 \cdot q = \frac{1}{64} \cdot 4 = \frac{1}{16}}$$

$$\begin{array}{r|l} 256 & 4 \\ 64 & 4 \\ 16 & 4 \\ 4 & 4 \\ 1 & \end{array}$$

Aplicatii

1) Sa se determine termenul de rang n al progresiei geometrice $\underset{b_1}{-1}, \underset{b_2}{\frac{1}{3}}, \underset{b_3}{-\frac{1}{9}}, \dots$

2) Sa se determine termenul de rang n al progresiei geometrice $\underset{b_1}{\sqrt{6}}, \underset{b_2}{\sqrt{3}}, \frac{\sqrt{6}}{2}, \dots$

1) $b_2 = b_1 \cdot q \Leftrightarrow \frac{1}{3} = (-1) \cdot q \Rightarrow \boxed{q = \frac{\frac{1}{3}}{-1} = -\frac{1}{3}}$

$$b_n = b_1 \cdot q^{n-1} \Leftrightarrow b_n = (-1) \cdot \left(-\frac{1}{3}\right)^{n-1}$$

2) $b_2 = b_1 \cdot q \Leftrightarrow \sqrt{3} = \sqrt{6} \cdot q \Leftrightarrow \boxed{q = \frac{\sqrt{3}}{\sqrt{6}} = \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}} \cdot \sqrt{2}} = \frac{1}{\sqrt{2}}}$

$$b_n = b_1 \cdot q \Leftrightarrow b_n = \sqrt{6} \cdot \left(\frac{1}{\sqrt{2}}\right)^{n-1}$$