Sapt. 9 (9-13 nov)

Logaritmarea unei expresii matematice

1) $\log_a A \cdot B = \log_a A + \log_a B$

2) $\log_a A : B = \log_a A - \log_a B$

3) $\log_a A^n = n \cdot \log_a A$

4) $\log_a A = \dfrac{\log_b A}{\log_b a}$

Ex. Sa se logaritmeze expresia $E = 5a^4 b^{\frac{5}{6}} c^3$.

$$\lg E = \lg\left(5a^4 b^{\frac{5}{6}} c^3\right) \overset{1)}{=} \lg 5 + \lg a^4 + \lg b^{\frac{5}{6}} + \lg c^3$$

$$\overset{3)}{=} \lg 5 + 4 \cdot \lg a + \frac{5}{6} \lg b + 3 \lg c$$

Ex. $E = (8a^2 b^4) : (3x\sqrt{y})$

$$\lg E = \lg\left[\underbrace{(8a^2 b^4)}_{A} : \underbrace{(3x\sqrt{y})}_{B}\right] \overset{2)}{=} \lg\underbrace{(8a^2 b^4)}_{A} - \lg\underbrace{(3x\sqrt{y})}_{B} = \lg 8 + \lg a^2 + \lg b^4 -$$

$$-\left(\lg 3 + \lg x + \lg\sqrt{y}\right) = \lg 8 + 2\lg a + 4\lg b - \lg 3 - \lg x - \lg y^{\frac{1}{2}} =$$

$$= \lg 8 + 2\lg a + 4\lg b - \lg 3 - \lg x - \frac{1}{2}\lg y$$

Ex. $E = \dfrac{7^2 \cdot \sqrt{94}}{\sqrt[3]{329 \sqrt[3]{4}}}$

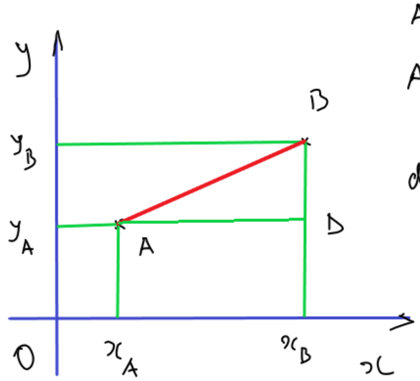
$$\lg E = \lg \frac{7^2 \cdot \sqrt{94}}{\sqrt[3]{329 \cdot \sqrt[3]{4}}} = \lg\left(7^2 \cdot \sqrt{94}\right) - \lg\left(\sqrt[3]{329 \cdot \sqrt[3]{4}}\right) =$$

$$= \lg 7^2 + \lg \sqrt{94} - \left(\lg\left(329 \cdot 4^{\frac{1}{3}}\right)^{\frac{1}{3}} = 2\lg 7 + \lg 94^{\frac{1}{2}} - \frac{1}{3}\lg\left(329 \cdot 4^{\frac{1}{3}}\right) =$$

$$= 2\lg 7 + \frac{1}{2}\lg 94 - \frac{1}{3}\left(\lg 329 + \lg 4^{\frac{1}{3}}\right) = 2\lg 7 + \frac{1}{2}\lg 94 - \frac{1}{3}\lg 329 - \frac{1}{9}\lg 4$$

Valorile $\lg 7, \lg 94, \lg 329, \lg 4$ pot fi luate din tabele de logaritm si astfel expresia complicata de la inceput poate fi calculata simplu doar cu adunari si scaderi.

Distanta dintre doua puncte din plan $A(x_A, y_A)$, $B(x_B, y_B)$

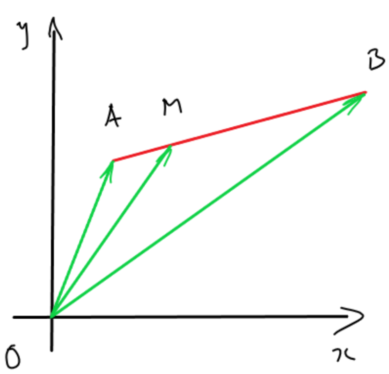


$$AB^2 = AD^2 + BD^2$$

$$AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2$$

$$\text{dist}(A,B) = \boxed{AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}}$$

Coordonatele unui punct care imparte segment in raport dat



$$\vec{AM} = k \cdot \vec{MB}, \quad \vec{OM} = \frac{\vec{OA} + k \cdot \vec{OB}}{1+k} = \frac{1}{1+k}\vec{OA} + \frac{k}{1+k} \cdot \vec{OB}$$

$$\vec{OA} = x_A \cdot \vec{i} + y_A \cdot \vec{j}$$

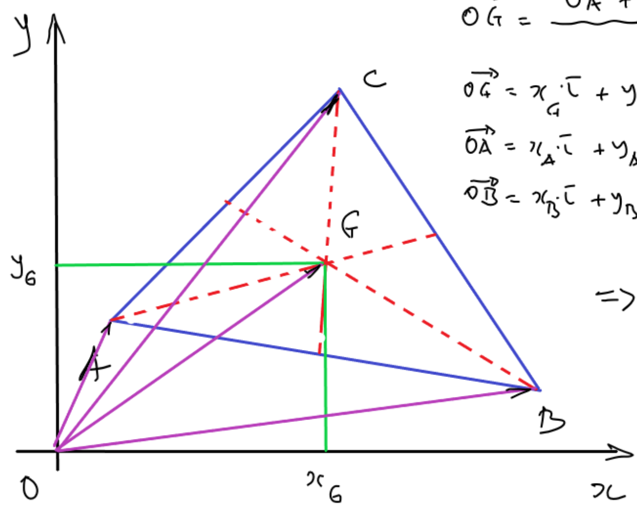
$$\vec{OB} = x_B \cdot \vec{i} + y_B \cdot \vec{j} \Rightarrow \vec{OM} = \frac{1}{1+k}\left(x_A\vec{i} + y_A\vec{j}\right) + \frac{k}{1+k}\left(x_B\vec{i} + y_B\vec{j}\right)$$

$$\vec{OM} = x_M \cdot \vec{i} + y_M \cdot \vec{j}$$

$$\Rightarrow x_M\vec{i} + y_M\vec{j} = \left(\frac{1}{1+k}x_A + \frac{k}{1+k}x_B\right)\vec{i} + \left(\frac{1}{1+k}y_A + \frac{k}{1+k}y_B\right)\vec{j}$$

$$\Rightarrow \begin{cases} x_M = \dfrac{1}{1+k} \cdot x_A + \dfrac{k}{1+k} \cdot x_B \\ y_M = \dfrac{1}{1+k} \cdot y_A + \dfrac{k}{1+k} \cdot y_B \end{cases}$$

Centrul de greutate al unui triunghi



$$\vec{OG} = \frac{\vec{OA} + \vec{OB} + \vec{OC}}{3}$$

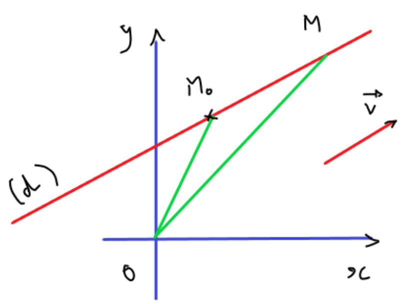
$$\vec{OG} = x_G \cdot \vec{i} + y_G \cdot \vec{j}, \quad \vec{OC} = x_C \cdot \vec{i} + y_C \cdot \vec{j}$$

$$\vec{OA} = x_A \cdot \vec{i} + y_A \cdot \vec{j}$$

$$\vec{OB} = x_B \cdot \vec{i} + y_B \cdot \vec{j}$$

$$\Rightarrow \begin{cases} x_G = \dfrac{x_A + x_B + x_C}{3} \\[2ex] y_G = \dfrac{y_A + y_B + y_C}{3} \end{cases}$$

Ecuatia dreptei determinata de un punct si o directie data $(M_0, \vec{v})$



$$M_0(x_0, y_0),\ M(x, y),\ \vec{v} = (a, b)$$

$$\vec{OM} = \vec{OM_0} + \vec{M_0M} = \vec{OM_0} + \alpha \cdot \vec{v}$$

$$(x, y) = (x_0, y_0) + \alpha (a, b)$$

$$\begin{cases} x = x_0 + \alpha a \\ y = y_0 + \alpha b \end{cases} \Rightarrow \begin{cases} x - x_0 = \alpha a \\ y - y_0 = \alpha b \end{cases}$$

(ecuatiile parametrice)

$$\Rightarrow \frac{x - x_0}{a} = \frac{y - y_0}{b} = \alpha \Rightarrow (x - x_0) b = (y - y_0) a \Leftrightarrow x \overset{A}{b} + y \cdot \overset{B}{(-a)} + \overbrace{y_0 a - x_0 b}^{C} = 0$$

(ecuatia carteziana)

$$\Rightarrow Ax + By + C = 0$$

(ecuatia generala)

Sapt. 10 (16-20 nov)

Aplicatie

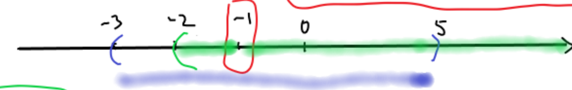
Sa se determine x numar real pentru care sunt definiti logaritmii

$\log_a A = b \Leftrightarrow a^b = A$

$0 < a \neq 1,\ A > 0$

a) $\log_{x+2}(15 + 2x - x^2)$

b) $\log_{4-x}(x^2 - x - 6)$



a) $\begin{cases} x+2 > 0 \\ x+2 \neq 1 \\ 15 + 2x - x^2 > 0 \end{cases} \Leftrightarrow \begin{cases} \boxed{x > -2} \\ \boxed{x \neq -1} \\ x \in (-3, 5) \end{cases} \Rightarrow x \in (-2, 5) \setminus \{-1\}$

$-x^2 + 2x + 15 = 0$

$\Delta = b^2 - 4ac = 4 - 4 \cdot (-1) \cdot 15 = 64$

$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-2 \pm 8}{-2} = \begin{matrix} -3 \\ 5 \end{matrix}$

x		-3		5	
$-x^2+2x+15$	$- - -$	0	$+ + +$	0	$- - -$

b) $\log_{4-x}(x^2 - x - 6)$

$\begin{cases} 4-x > 0 \\ 4-x \neq 1 \\ x^2 - x - 6 > 0 \end{cases} \Leftrightarrow \begin{cases} 4 > x \\ 3 \neq x \\ x \in (-\infty, -2) \cup (3, +\infty) \end{cases} \Leftrightarrow \begin{cases} x \in (-\infty, 4) \\ x \neq 3 \\ x \in (-\infty, -2) \cup (3, +\infty) \end{cases}$

$x^2 - x - 6 = 0$

$\Leftrightarrow x \in (-\infty, -2) \cup (3, 4)$

$\Delta = b^2 - 4ac = (-1)^2 - 4 \cdot 1 \cdot (-6) = 25$

$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{1 \pm 5}{2} = \begin{matrix} 3 \\ -2 \end{matrix}$

x	$-\infty$	-2		3	$+\infty$
x^2-x-6	$+ +$	0	$- - -$	0	$+ + +$

Logaritmarea unei expresii matematice

Ex. Sa se logaritmeze expresia $E = 5a^4 b^{\frac{5}{6}} c^3$.

$$\lg E = \lg\left(5a^4 b^{\frac{5}{6}} c^3\right) \overset{1)}{=} \lg 5 + \lg a^4 + \lg b^{\frac{5}{6}} + \lg c^3$$

$$\overset{3)}{=} \lg 5 + 4\cdot \lg a + \frac{5}{6}\lg b + 3\lg c$$

1) $\log_a A\cdot B = \log_a A + \log_a B$

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$$\lg E = \lg\left[\underbrace{(8a^2b^4)}_{A} : \underbrace{(3x\sqrt{y})}_{B}\right] \overset{2)}{=} \lg(\underbrace{8a^2b^4}_{A}) - \lg(\underbrace{3x\sqrt{y}}_{B}) = \lg 8 + \lg a^2 + \lg b^4 -$$

$$-\left(\lg 3 + \lg x + \lg\sqrt{y}\right) = \lg 8 + 2\lg a + 4\lg b - \lg 3 - \lg x - \lg y^{\frac{1}{2}} =$$

$$= \lg 8 + 2\lg a + 4\lg b - \lg 3 - \lg x - \frac{1}{2}\lg y$$

Ex. $E = \dfrac{7^2\cdot\sqrt{94}}{\sqrt[3]{329\sqrt[3]{4}}}$

$$\lg E = \lg\frac{7^2\cdot\sqrt{94}}{\sqrt[3]{329\cdot\sqrt[3]{4}}} = \lg\left(7^2\cdot\sqrt{94}\right) - \lg\left(\sqrt[3]{329\cdot\sqrt[3]{4}}\right) =$$

$$= \lg 7^2 + \log\sqrt{94} - \left(\lg\left(329\cdot 4^{\frac{1}{3}}\right)^{\frac{1}{3}}\right) = 2\lg 7 + \lg 94^{\frac{1}{2}} - \frac{1}{3}\lg\left(329\cdot 4^{\frac{1}{3}}\right) =$$

$$= 2\lg 7 + \frac{1}{2}\lg 94 - \frac{1}{3}\left(\lg 329 + \lg 4^{\frac{1}{3}}\right) = 2\lg 7 + \frac{1}{2}\lg 94 - \frac{1}{3}\lg 329 - \frac{1}{9}\lg 4$$

Valorile $\lg 7, \log 94, \log 329, \lg 4$ pot fi luate din tabele de logaritm si astfel expresia complicata de la inceput poate fi calculata simplu doar cu adunari si scaderi.

$\lg 7 = 0{,}84\ldots$ $\quad$ $\lg 329 = 2{,}51\ldots$

$\lg 97 = 1{,}97\ldots$ $\quad$ $\lg 4 = 0{,}60\ldots$

$$\lg E \approx 2\cdot 0{,}84 + 0{,}5\cdot 1{,}97 - 0{,}33\cdot 2{,}51 - 0{,}11\cdot 0{,}60 \approx 1{,}64 +$$

$$+ 0{,}99 - 0{,}82 - 0{,}06 = 1{,}75$$

$$E = 10^{1{,}75} \Rightarrow \boxed{E \approx 56{,}23}$$

Aplicatie

Sa se logaritmeze expresia $\left(a^2\cdot\sqrt[4]{b^2}\right):\left(b^2\cdot\sqrt[3]{a}\right)$

$$\ln E = \ln\left[\left(a^2\cdot\sqrt[4]{b^2}\right):\left(b^2\cdot\sqrt[3]{a}\right)\right] = \ln\left(a^2\cdot\sqrt[4]{b^2}\right) - \ln\left(b^2\cdot\sqrt[3]{a}\right) =$$
$$= \ln a^2 + \ln\sqrt[4]{b^2} - \left(\ln b^2 + \ln\sqrt[3]{a}\right) = 2\ln a + \ln b^{\frac{2}{4}} - 2\ln b - \ln a^{\frac{1}{3}}$$
$$= 2\ln a + \frac{2}{4}\ln b - 2\ln b - \frac{1}{3}\ln a = \left(\overset{3)}{2} - \frac{1}{3}\right)\cdot\ln a + \left(\frac{2}{4} - 2\right)\cdot\ln b =$$
$$= \frac{5}{3}\ln a - \frac{3}{2}\ln b$$

ln - logaritm natural (logaritm in baza e) $\boxed{\text{e = 2,7... constanta lui Euler}}$, $\ln e = \log_e e = 1$

Aplicatie

$\lg 2 = 0{,}30\ldots$
$\lg 5 = 0{,}69\ldots$

$$E = \frac{512^{0,75}}{125^{\sqrt{5}}}\cdot 25^{3,25} : \sqrt[15]{625}$$

$$\lg E = \lg\frac{512^{0,75}}{125^{\sqrt{5}}} + \lg 25^{3,25} - \lg\sqrt[15]{625} = \lg 512^{0,75} - \lg 125^{\sqrt{5}} + 3{,}25\cdot\lg 25 -$$
$$- \lg 625^{\frac{1}{15}} = 0{,}75\cdot\lg 512 - \sqrt{5}\cdot\lg 125 + 3{,}25\cdot\lg 25 - \frac{1}{15}\lg 625 =$$
$$= 0{,}75\cdot\lg 2^9 - \sqrt{5}\cdot\lg 5^3 + 3{,}25\cdot\lg 5^2 - \frac{1}{15}\lg 5^4 = 0{,}75\cdot 9\cdot\lg 2 - 3\sqrt{3}\lg 5 +$$
$$+ 2\cdot 3{,}25\lg 5 - \frac{4}{15}\lg 5 \approx 0{,}75\cdot 9\cdot 0{,}3 - 3\cdot\sqrt{3}\cdot 0{,}69 - 2\cdot 3{,}25\cdot 0{,}69 - \frac{4}{15}\cdot 0{,}69 \approx$$
$$\approx 2{,}02 - 7{,}41 - 4{,}48 - 0{,}18 \approx -10{,}05$$

Tema
Din manual, pag.35, E7

$$\boxed{\lg E \approx -10{,}05} \Rightarrow \boxed{E = 10^{-10,05} = \frac{1}{10^{10,05}} \approx 8{,}91\cdot 10^{-11}}$$

Sapt. 11 (23-27 nov)

Aplicatie

$\lg 8 \approx 0,68124$, $\lg 3 \approx 0,63647$ / $\lg 24 = ?$, $\lg 0,03 = ?$, $\lg 80 = ?$

$\lg 24 = \lg(8\cdot 3) = \lg 8 + \lg 3 = 0,68124 + 0,63647 = 1,31771$

$\lg 0,03 = \lg(3:100) = \lg 3 - \lg 100 = \lg 3 - 2\lg 10 = 0,63647 - 2 = -1,36353$

$\lg 80 = \lg(8\cdot 10) = \lg 8 + \lg 10 = 0,68124 + 1 = 1,68124$

$$\boxed{\begin{aligned}\log_a(A\cdot B) &= \log_a A + \log_a B\\ \log_a(A:B) &= \log_a A - \log_a B\end{aligned}}$$

Aplicatie

a) $\log_{\frac{1}{\sqrt5}}\sqrt{125}$ $\underline{\underline{\text{trecem}}}$ la baza 5 $\dfrac{\log_5\sqrt{125}}{\log_5\frac{1}{\sqrt5}} = \dfrac{\log_5(125)^{\frac12}}{\log_5(\sqrt5)^{-1}} = \dfrac{\frac12\log_5 125}{-\log_5\sqrt5} = -\dfrac12\,\dfrac{\log_5 5^3}{\log_5 5^{\frac12}} =$

$= -\dfrac12\,\dfrac{3\cdot\log_5 5}{\frac12\log_5 5} = -\cancel{\dfrac12}\cdot\dfrac{3}{\cancel{\frac12}} = -3$

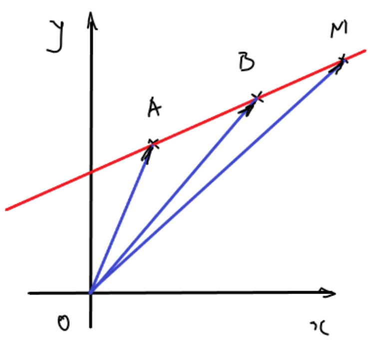
$$\boxed{\log_a A = \frac{\log_b A}{\log_b a}} \qquad \boxed{\log_a A^n = n\cdot\log_a A} \qquad \boxed{\log_a a = 1}$$

b) $\log_{\sqrt{2}}\left(\log_{\sqrt{5}} 625\right) = \log_{\sqrt{2}}(8) = \frac{\log_2 8}{\log_2 \sqrt{2}} = \frac{\log_2 2^3}{\log_2 2^{\frac{1}{2}}} = \frac{3 \cdot \log_2 2}{\frac{1}{2} \cdot \log_2 2} = \frac{3}{\frac{1}{2}} = 6$

$\log_{\sqrt{5}} 625 = \frac{\log_5 625}{\log_5 \sqrt{5}} =$

$= \frac{\log_5 5^4}{\log_5 5^{\frac{1}{2}}} = \frac{4 \log_5 5}{\frac{1}{2} \log_5 5} = \frac{4}{\frac{1}{2}} = 8$

Ecuatia dreptei determinate de doua puncte



$\vec{OM} = \vec{OA} + \alpha \cdot \vec{AB}, \quad \alpha \in \mathbb{R}$

$(x_M, y_M) = (x_A, y_A) + \alpha \cdot (x_B - x_A, \; y_B - y_A)$

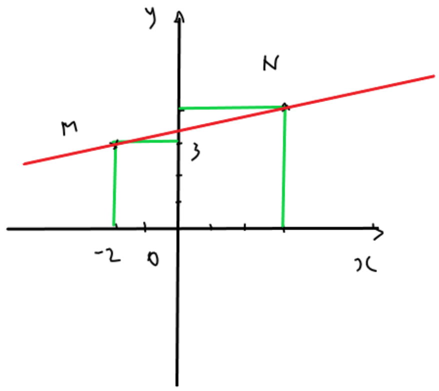
$$\begin{cases} x_M = x_A + \alpha(x_B - x_A) \\ y_M = y_A + \alpha(y_B - y_A) \end{cases}$$

$\frac{x_M - x_A}{x_B - x_A} = \frac{y_M - y_A}{y_B - y_A} = \alpha, \quad \forall M \in AB$

$\frac{x - x_A}{x_B - x_A} = \frac{y - y_A}{y_B - y_A}$ (ecuatia dreptei prin doua puncte) A , B

Aplicatie

Sa se determine ecuatia dreptei ce trece prin punctele M(-2,3), N(3,4) si sa se reprezinte grafic.



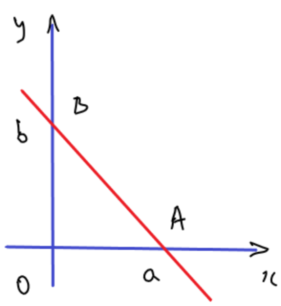
$$\frac{x-x_M}{x_N-x_M}=\frac{y-y_M}{y_N-y_M} \Leftrightarrow \frac{x-(-2)}{3-(-2)}=\frac{y-3}{4-3} \Leftrightarrow$$

$$\Leftrightarrow \frac{x+2}{5}=\frac{y-3}{1} \Leftrightarrow x+2=5(y-3) \Leftrightarrow$$

$$\Leftrightarrow x-5y+2+15=0 \Leftrightarrow \boxed{x-5y+17=0}$$

Caz particular ecuatia dreptei prin taieturi

$A(a,0)$, $B(0,b)$



$$\frac{x-x_A}{x_B-x_A}=\frac{y-y_A}{y_B-y_A} \Leftrightarrow \frac{x-a}{0-a}=\frac{y-0}{b-0} \Leftrightarrow$$

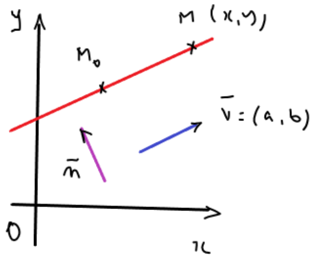
$$\Leftrightarrow (x-a)\cdot b = y\cdot(-a) \Leftrightarrow xb-ab=-ya \Leftrightarrow$$

$$\Leftrightarrow xb+ya=ab \;\Big|\; :ab$$

$$\Leftrightarrow \boxed{\frac{x}{a}+\frac{y}{b}=1}$$

(ecuatia dreptei prin taieturi)

Observatii



$\alpha \in R$

$$\begin{cases} x = x_0 + \alpha \cdot a \\ y = y_0 + \alpha \cdot b \end{cases} \Leftrightarrow \begin{cases} x - x_0 = \alpha \cdot a \\ y - y_0 = \alpha \cdot b \end{cases} \Rightarrow \frac{x-x_0}{a} = \frac{y-y_0}{b}$$

$$x \cdot b - x_0 \cdot b = y \cdot a - y_0 \cdot a$$

$$x \cdot b + y \cdot (-a) + y_0 \cdot a - x_0 \cdot b = 0$$

$x \cdot A + y \cdot B + C = 0$ (ec. generala)

$\bar{v} = (a,b) = (-B, A)$ (vector director)

$\bar{n} = (b,-a) = (A, B)$ (vector normal la dreapta)

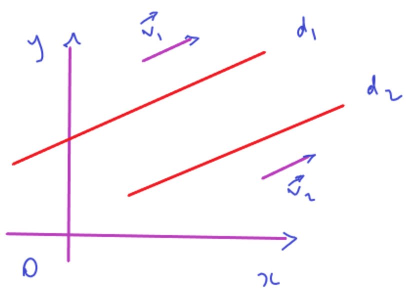
m = coeficient unghiular

$$y = -\frac{A}{B}x - \frac{C}{B}$$

n = ordonata la origine

$y = mx + n$ (ecuatia explicita a dreptei)

Conditii de paralelism a doua drepte



$d_1: a_1x + b_1y + c_1 = 0$, $\vec{v}_1 = (-b_1, a_1)$

$d_2: a_2x + b_2y + c_2 = 0$, $\vec{v}_2 = (-b_2, a_2)$

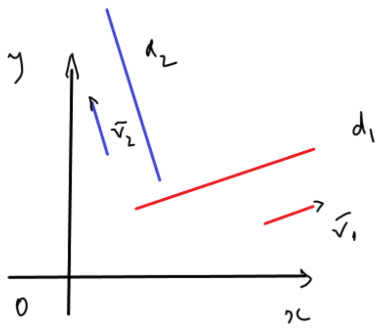
$$\vec{v}_1 \parallel \vec{v}_2 \Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \Leftrightarrow d_1 \parallel d_2$$

$d_1: y = m_1x + n_1$

$d_2: y = m_2x + n_2$

$$d_1 \parallel d_2 \Leftrightarrow m_1 = m_2$$

Conditii de perpendicularitate a doua drepte



$d_1: \ a_1x + b_1y + c_1 = 0 \quad , \vec{v_1} = (-b_1, a_1)$

$d_2: \ a_2x + b_2y + c_2 = 0 \quad , \vec{v_2} = (-b_2, a_2)$

$d_1 \perp d_2 \iff \bar{v}_1 \perp \bar{v}_2 \iff \bar{v}_1 \cdot \bar{v}_2 = 0 \iff$

$\iff a_1a_2 + b_1b_2 = 0$

$d_1: \ y = m_1x + n_1$

$d_2: \ y = m_2x + n_2$

$d_1 \perp d_2 \iff m_1 \cdot m_2 = -1$

Tema: pag. 280, E1,E2